Skill Accumulation in the Market and at Home

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Online Appendix

A Decentralized Economy

This section decentralizes the planner's problem outlined in Section 3.1. Within a period, timing in the decentralized economy is similar to that described for the planner, with the following changes. First, employed workers may choose to separate from their match with probability $d \in [\delta, 1]$. Second, submarkets are indexed by (x, r, s, ψ), where $x \in \mathbb{R}$ is the value in terms of the worker's lifetime utility of the match, $r \in [\underline{y}, \overline{y}]$ is the cutoff for match specific productivity above which the worker accepts the job, and $s = (z, h)$ is the skill pair of the worker for which the vacancy is intended. Second, in the production stage, employed worker-firm pairs produce Azy and employed workers consume their labor income w. In the search stage, firms choose submarkets in which to post vacancies and workers observe the distribution of offers before choosing one submarket in which to search. A firm may post a vacancy by paying a cost $k > 0$. Within a submarket, each vacancy offers the same value x, and firms commit to this value as well as the type of worker they will hire if a match occurs.[1]

At the beginning of the production stage, the value function for an unemployed worker of type $s = (z, h)$ is

$$V_U(s, \psi) = \sup_{x,r} \left\{ h + \beta(1-\xi) \left[\mathbb{E}_U(V_U(s', \hat{\psi})) + D(x, r, s, \mathbb{E}_U V_U(s', \hat{\psi}), \hat{\psi}) \right] + \beta\xi \mathbb{E}_0(V_U(s', \hat{\psi})) \right\} \quad \text{(A1)}$$

where

$$D(x, r, s, V, \psi) = p(\theta(x, r, s, \psi)) \overline{F}(r) \big(x - V \big)$$

and $\overline{F}(r) = \int_r^{\overline{y}} f(y)\, dy$. The policy functions are denoted $(x_u(s, \psi), r_u(s, \psi))$ and the implied market tightness is denoted $\theta(x, r, s, \psi)$.

Given a wage w, the value function for an employed worker with $s = (z, h)$ and match

[1] In this model with worker-specific skills, workers do not necessarily sort across submarkets endogenously, as they do in Menzio and Shi (2011). To guarantee that each submarket contains only one worker type, I assume firm commitment to worker skills, which are assumed to be observable.

productivity y at the beginning of the production stage is given by

$$V_E(s,y,\psi;w) = \max_{x,r,d}\Big\{w + \beta(1-\xi)\Big(d\mathbb{E}_E(V_U(s',\hat{\psi})) + (1-d)\mathbb{E}_E(V_E(s',y,\hat{\psi})) + \lambda_e D(x,r,s,\mathbb{E}_E(V_E(s',y,\hat{\psi})),\hat{\psi})\Big) + \beta\xi\mathbb{E}_0(V_U(s',\hat{\psi}))\Big\} \quad \text{(A2)}$$

Similarly, the value of a firm with match-specific productivity y, employing a worker whose current skills are (z,h) given her wage w and policy functions can be written

$$J(s,y,\psi;w) = Azy - w + \beta(1-\xi)(1-d)[1-\lambda_e p(\theta(x,r,s,\psi))\overline{F}(r)]\mathbb{E}_E(J(s',y,\hat{\psi};w')) \quad \text{(A3)}$$

I assume that employment contracts are complete in the sense that they specify the wage and separation probability as a function of tenure t and history of productivities $\{s^t;y;A^t\}$ in the match. As shown in Menzio and Shi (2011) and discussed in earlier related work by Moen and Rosén (2004), this contractual environment results in bilaterally efficient contracts which maximize the sum of the firm's expected profits and the worker's expected utility. This result follows from the fact that firms must guarantee the expected value x to any worker with whom it matches, forcing the firm to internalize the optimal choices of the worker when choosing the contract. Hence the optimal choice of the separation probability, productivity cutoff, and match value maximizes the value of the match, which is equal to the sum of (A2) and (A3):

$$V_M(s,y,\psi) = \sup_{x,r,d}\Big\{Azy + \beta(1-\xi)\big[d\mathbb{E}_E V_U(s',\hat{\psi}) + (1-d)\big(\lambda_e p(\theta)\overline{F}(r)x + (1-\lambda_e p(\theta)\overline{F}(r))\mathbb{E}_E(V_M(s',y,\hat{\psi}))\big)\big] + \beta\xi\mathbb{E}_0(V_U(s',\hat{\psi}))\Big\} \quad \text{(A4)}$$

The wage is absent from (A4) because it is a transfer from the firm to the worker, leaving the value of the match unchanged. The policy functions are denoted $(x_e(s,y,\psi), r_e(s,y,\psi))$.

To close the model, there is free entry into vacancy posting in every submarket,[2] so that the firm's benefit of vacancy creation in a non-empty submarket (x,r,s,ψ) is equal to the cost:

$$k \geq \beta(1-\xi)q(\theta(x_i,r_i,s,\psi))\left(\int_{r_i}^{\overline{y}} \mathbb{E}_i(V_M(s',y',\hat{\psi}))\ f(y')dy' - x_i\right), \quad i = U,E \quad \text{(A5)}$$

and $\theta(x_i,r_i,s,\psi) \geq 0$ with complementary slackness. Since the timing of the model is

[2] From (A5), it is clear that the present model is not equivalent to a model with one relative skill, i.e. z/h. It is the constant cost of vacancy posting that breaks the model's homogeneity. Appendix F extends the model to one with a proportional vacancy cost and proves that the extended model is indeed homogeneous in h. Appendix F shows that the intuition driving the fall in individual job-finding probabilities breaks down. As workers lose relative skills, their productivity falls together with the cost for firms to post vacancies targeting these workers. Thus, in general the job-finding probability will not fall sharply with duration as it does in the two-skill model due to this vacancy cost-production trade-off.

such that matches are made at the end of the period, a firm offers lifetime utility x which is known one period before production takes place. Therefore the firm discounts the expected value of the match by $\beta(1-\xi)$. The expectation is taken with respect to the conditional distribution of productivities while unemployed since the worker evolves once more before beginning production. This distribution depends only on the current type of the worker that the firm commits to hire, and not on the entire distribution of workers across types. Note that although the firm knows the worker's current type, because production occurs after one period, there is uncertainty about the worker's type when production actually takes place. In this sense, the worker's skills are an "experience good" in the terminology of Menzio and Shi (2011).

Since each firm posting a vacancy for value x commits to hire a single type, the firm knows for certain the current type of worker that it will hire in any submarket. For any worker of a type different than s, it is not optimal to search in submarket (x, r, s, ψ) since there is probability zero that she will be hired. Thus, the firm's decision to post a vacancy does not depend on the distribution of searching workers. Zero expected profits in equilibrium imply that firms are indifferent regarding the submarket in which to post vacancies.

Following the literature, equilibria are restricted to those in which the market tightness satisfies complementary slackness condition (A5) in every submarket. This implies that firms must be indifferent between posting vacancies in any submarket, whether or not it is active in equilibrium, so that market tightness is always pinned down by the free entry condition. I now turn to the definition of equilibrium.

Definition 1. *A block recursive equilibrium (BRE) consists of a market tightness function $\theta : \mathbb{R}\times Y\times S\times \mathbf{A} \to \mathbb{R}_+$, a value function for the unemployed worker $V_U : Z\times H\times \mathbf{A} \to \mathbb{R}$, policy functions for the unemployed worker $x_u : S\times \mathbf{A} \to \mathbb{R}$ and $r_u : S\times \mathbf{A} \to Y$, a value function for the employed worker $V_E : Z\times H\times Y\times \mathbf{A} \to \mathbb{R}$, policy functions for the employed worker $x_e : S\times Y\times \mathbf{A} \to \mathbb{R}$, $r_e : S\times Y\times \mathbf{A} \to Y$, and $d : S\times Y\times \mathbf{A} \to [\delta, 1]$ where:*

- *(i) $V_U(z,h,A)$ satisfies (A1) $\forall(z,h,\psi)\in Z\times H\times \Psi$ and $(x_u(s,A), r_u(s,A))$ are the associated policy functions.*
- *(ii) $V_M(z,h,y,A)$ satisfies (A4) $\forall(z,h,y,\psi)\in Z\times H\times Y\times\Psi$ and $x_e(s,y,A), r_e(s,y,A)$, and $d(s,y,A)$ are the associated policy functions.*
- *(iii) $\theta(x,r,s,A)$ satisfies (A5) $\forall(x,r,s,\psi)\in\mathbb{R}\times Y\times S\times\Psi$*

In any BRE, agents' value and policy functions are independent of the distributions of workers. Given the market tightness function θ, condition (i) ensures that unemployed workers' search strategies are optimal and condition (ii) ensures that employed worker-firm pairs' search and separation strategies are optimal. Condition (iii) states that the market tightness function θ is consistent with firms' incentives to create vacancies.

Given the infinite horizon programming problem faced by individuals in the decentralized economy, the analysis of equilibrium proceeds as follows. First, a lemma is stated showing that there exists a functional equation for all agents equivalent to solving equilibrium conditions (A1), (A4), and (A5). Then, Theorem 2 shows that the functional equation from the lemma satisfies boundedness and continuity restrictions and therefore admits a unique solution. Further, by the recursive structure of the functional equation, the solutions of the problem are independent of the distributions (u, e) and satisfy Definition 1, therefore the unique decentralized equilibrium is a BRE.

Lemma 1. *An equilibrium exists if and only if it solves the following problem:*

$$V(\alpha, z, h, y, \psi) = \alpha\Bigg(\max_{\theta\in[0,\overline{\theta}], d\in[\delta,1]} \Bigg\{ Azy - k\lambda_e\theta(1-d) + \beta(1-\xi)\big[d\mathbb{E}_E V(0, z, h, y, \psi)$$
$$+(1-d)\big(\mathbb{E}_E V(1, z', h', y, \psi) + \int_Y \max_{c\in[0,1]} \big\{\lambda_e p(\theta) c\mathbb{E}_E\big(V(1, z', h', y', \psi) - V(1, z', h', y, \psi)\big)\big\} f(y')\, dy'\big)\big] \Bigg\} \Bigg)$$
$$+ (1-\alpha) \max_{\theta\in[0,\overline{\theta}]} \big\{ h - k\theta + \beta(1-\xi)\big(\mathbb{E}_U(V(0, z', h', y, \hat{\psi}))$$
$$+ \int_Y \max_{c\in[0,1]} \{p(\theta)c\mathbb{E}_U(V(1, z', h', y'\hat{\psi}) - V(0, z', h', y, \hat{\psi}))'\} f(y')dy\big)\big\} + \beta\xi\mathbb{E}_0(V(0, z', h', y, \hat{\psi})) \tag{A6}$$

$$\textit{where } V(0, z, h, y, \psi) \equiv V_U(z, h, \psi), \quad V(1, z, h, y, \psi) \equiv V_M(z, h, y, \psi)$$

and the period payoff function, $\alpha(Azy - k\lambda_e\theta(1-d)) + (1-\alpha)(h - k\theta)$, *is bounded and continuous.*

The upper bound for market tightness, $\overline{\theta} < \infty$, is given by the larger of $\overline{\theta_u}$ and $\overline{\theta_e}$ in part (ii) of the proof of Theorem 1. Using the lemma, it is straightforward to prove the following theorem.

Theorem 2. *(i) All equilibria are block recursive. (ii) There exists a unique BRE.*

Part (i) of Theorem 2 comes from the assumptions of directed search and complete contracts. Given a fixed aggregate productivity A, if there are two submarkets committed to hire a worker of type s, the worker faces a trade-off between a higher probability of matching and a higher expected value of the match. The higher is the value offered in a submarket committed to s, the more applicants of type s relative to vacancies it will attract, decreasing the probability for an individual worker to find a match. Since the firm commits to hire a certain type of worker, the firm's probability of matching will depend only on one worker type rather than the distribution of searching workers across productivities.

The existence of type-specific submarkets acts to complete the labor market in the sense that market tightness is specific to each productivity pair and therefore provides a "price" for each type. The contracting assumption along with firm commitment allows me to restrict attention to the value of a match and pin down the lifetime value to the employed worker, x, as a function of the market tightness and the match value. Due to the two-dimensional heterogeneity of workers, without the restriction of commitment it is possible that two types of workers will find it optimal to search in the same submarket, causing the block recursive property of the equilibrium to break down. In this case, equilibria will still exist, although they will not be explored here.

A.1 Efficiency of the Decentralized Equilibrium

The following proposition states that the equilibrium described in the section above is efficient in the sense that the value and policy functions satisfying the BRE are identical to those that solve the planner's problem discussed in Section 3.

Proposition 1. *The unique BRE in the decentralized economy is efficient, that is,* $\theta(x_e(s,y,A), r_e(s,y,A), s, A) = \theta^*_e(s,y,A)$, $\theta(x_u(s,A), r_u(s,A), s, A) = \theta^*_u(s,A)$, $r_e(s,y,A) = r^*_e(s,y,A)$, $r_u(s,A) = r^*_u(s,A)$, *and* $d(s,y,A) = d^*(s,y,A)$.

The decentralized equilibrium is efficient because of the presence of type-specific submarkets and the assumptions of complete contracts and firm commitment to types. As discussed regarding Theorem 2, the presence of submarkets in which only one worker type searches causes firms to internalize the externalities that are typically present in other models. Since the planner values home productivity as much as workers in the decentralized economy, it is easy to show that the planner's value of unemployment, $W_U(z,h,A)$, satisfies (A1).

Without complete contracts, a firm and worker would not necessarily divide the surplus optimally, and the joint value function (A4) would not be solved by the value of a match to the planner $W_E(z,h,y,A)$. However, when the surplus is maximized by restricting the contract space, it can be shown that the value of a match in the decentralized equilibrium and the value of an employed worker to the planner both solve (A4).

B Proofs

B.1 Proof of Theorem 1

(i) This part of the proof shows that any solution to (3) also solves (6) and vice versa, proving the equivalence of the two formulations.

To simplify notation, I will write

$$\mathbb{E}_U(W_U(z',h',\hat{A})) = \mathbb{E}_U(W_U(z',h',\hat{A})|z,h,A),$$

$$\mathbb{E}_E(W_E(z',h',y,\hat{A})) = \mathbb{E}_E(W_E(z',h',y,\hat{A})|z,h,y,A),$$
$$\mathbb{E}_0(W_U(z',h',\hat{A})) = \mathbb{E}_0(W_U(z',h',\hat{A})|A).$$

Denote the set of solutions to (3) as $\mathcal{A}_1$ and to (6) as $\mathcal{A}_2$.

Suppose $\big(c^*_u(z,h,y,A), c^*_e(z,h,y,y',A), \theta^*_e(z,h,y,A), \theta^*(z,h,A), d^*(z,h,y,A), W_U(z,h,A),$

$W_E(z,h,y,A)\big) \in \mathcal{A}_2$. Plugging in for the laws of motion $\hat{u}$ and $\hat{e}$,

$$\tilde{W}^*(\psi) = \int_Z \int_H \Bigg(C(d^*, \theta_u^*, \theta_e^*, A) + \beta(1-\xi) \Big[\mathbb{E}_U(W_U(z',h',\hat{A}))$$
$$+ p(\theta_u^*(z,h,A)) \int_Y c_u^*(z,h,y',A) \mathbb{E}_U \big(W_E(z',h',y',\hat{A}) - W_U(z',h',\hat{A}) \big) f(y') dy' \Big] u(z,h)$$
$$+ \beta(1-\xi) \int_Y \Bigg(d^*(z,h,y,\hat{A}) \mathbb{E}_E W_U(z',h',\hat{A}) + (1 - d^*(z,h,y,\hat{A})) \Big[\mathbb{E}_E W_E(z',h',y,\hat{A})$$
$$+ \lambda_e p(\theta_e^*(z,h,y,A)) \int_Y c_e^*(z,h,y,y',A) \mathbb{E}_E \big(W_E(z',h',y',\hat{A}) - W_E(z',h',y,\hat{A}) \big) f(y') dy' \Big] \Bigg) e(z,h,y)\, dy$$
$$+ \beta\xi \mathbb{E}_0(W_U(z',h',\hat{A}))(u(z,h) + \int_Y e(z,h,y)\ dy) \Bigg)\ dh\ dz$$

where

$$C(d^*, \theta_u^*, \theta_e^*, A) = \int_Z \int_H \big[(h - k\theta_u^*(z,h,A)) u(z,h)$$
$$+ \int_Y \big(Azy - k\lambda_e \theta_e^*(z,h,y,A)(1 - d^*(z,h,y,A)) \big) e(z,h,y)\ dy \big]\ dh\ dz$$

The integrals in the equation for $\tilde{W}^*(\psi)$ may be interchanged given the independence of (z,h) and A in Assumption 1. Imposing the definitions for the laws of motion of $\hat{u}$ and $\hat{e}$, one can rewrite $\tilde{W}^*(\psi)$ as

$$\begin{aligned}
\tilde{W}^*(\psi) &= C^*(d^*, \theta_u^*, \theta_e^*) + \beta E \int_Z \int_H \Bigg(W_U(z',h',\hat{A}) \hat{u}(z',h') \\
&\quad + \int_Y W_E(z',h',y',\hat{A}) \hat{e}(z',h',y')\ dy' \Bigg)\ dh'\ dz' \\
&= C^*(d^*, \theta_u^*, \theta_e^*) + \beta E \tilde{W}^*(\hat{\psi})
\end{aligned}$$

Since $\theta_u^*(z,h,A)$ and $c_u^*(z,h,y,A)$ solve $W_U(z,h,A)$ and $\theta_e^*(z,h,y,A)$, $c_e^*(z,h,y,y',A)$, and $d^*(z,h,y,A)$ solve $W_E(z,h,y,A)$, one cannot improve upon the welfare for any type because each type's value function was maximized separately in (6). Further, the policy functions must also maximize $\tilde{W}^*(\psi)$ since the integral of maxima is equal to the maximum of the integral when the problem of one type does not depend on the problem of any other type. Therefore the solution satisfying (6) must also satisfy (3), that is, $\mathcal{A}_2 \subseteq \mathcal{A}_1$. The proof of the converse ($\mathcal{A}_1 \subseteq \mathcal{A}_2$) is straightforward using the definitions of $\hat{u}$ and $\hat{e}$. It follows that $\mathcal{A}_2 = \mathcal{A}_1$.

(ii) This part of the proof shows the existence and uniqueness of the solution to (6). The optimality conditions for $W_U(z,h,A)$ imply that if $\theta_u^*(z,h,A) > 0$,

$$k = \beta(1-\xi) p'(\theta_u^*(z,h,A)) \int_Y c_u^*(z,h,y,A) \mathbb{E}_U \big[W_E(z',h',y,\hat{A}) - W_U(z',h',\hat{A}) \big] f(y) dy$$

Since the left hand side is strictly positive, $c_u^* \in [0,1]$, and $\mathbb{E}_U\big(W_E(z',h',y,\hat{A})) - W_U(z',h',\hat{A})\big) > 0$ whenever $\theta_u > 0$, $p'(\theta_u) \to 0$ as $\theta_u \to \infty$ implies that $\exists\ \overline{\theta_u} < \infty$ such that the optimality condition holds. Therefore the constraint set for θ_u is non-empty, compact, and continuous. Similarly, if $\theta_e^*(z,h,y,A) > 0$

$$\begin{aligned}
&k\lambda_e\theta_e(z,h,y,A)\\
&= \beta(1-\xi)\bigg(\int_Y \lambda_e p(\theta_e(z,h,y,A))c_e(z,h,y,y',A)\mathbb{E}_E\big[W_E(z',h',y',\hat{A}) - W_U(z',h',\hat{A})\big]f(y')dy'\\
&+ \int_Y [1-\lambda_e p(\theta_e(z,h,y,A))c_e(z,h,y,y',A)]\mathbb{E}_E\big[W_E(z',h',y,\hat{A}) - W_U(z',h',\hat{A})\big]f(y')dy'\bigg)
\end{aligned}$$

implies $\exists\ \overline{\theta_e} < \infty$ such that the optimality condition holds. Since θ_u^*, θ_e^*, c_u^*, c_e^* and d^* are bounded, current period utility is bounded and continuous, $\xi \in [0,1]$, and $\beta \in (0,1)$, solutions to $W_U(z,h,A)$ and $W_E(z,h,y,A)$ exist by Theorem 4.2 in Stokey, Lucas, and Prescott (1989) (henceforth, SLP). It is straightforward to show that Blackwell's sufficient conditions for a contraction hold, therefore the operator associated with $W_U(z,h,A)$ and $W_E(z,h,y,A)$ have unique solutions in the space of continuous bounded functions on $Z \times H \times \mathbf{A}$ and $Z \times H \times Y \times \mathbf{A}$, respectively.

Since all variables are bounded and $\mathbb{E}_i(W_E(z',h',y',\hat{A}))$, and $\mathbb{E}_i(W_U(z',h',\hat{A}))$ for $i = U, E$, and $\mathbb{E}_0(W_U(z',h',\hat{A}))$ are bounded functions of $W_E(z,h,y,A)$ and $W_U(z,h,A)$, the value functions are bounded. By Theorem 4.3 in SLP, the unique solution to the operators associated with $W_U(z,h,A)$ and $W_E(z,h,y,A)$ coincide with the solutions to

$$\begin{aligned}
&W_U(z,h,A) = \max_{\theta_u \in [0,\overline{\theta}]} \bigg\{ h - k\theta_u + \beta\Big[\xi\mathbb{E}_0[W_U(z',h',\hat{A})]\\
&+(1-\xi)\bigg(\int_Y \max_{c_u \in [0,1]} \big\{p(\theta_u)c_u\mathbb{E}_U[W_E(z',h',y,\hat{A})] + (1-p(\theta_u)c_u)\mathbb{E}_U[W_U(z',h',\hat{A})]\big\}f(y)dy\bigg)\Big]\bigg\}
\end{aligned}$$

and

$$\begin{aligned}
W_E(z,h,y,A) = &\max_{\theta_e \in [0,\overline{\theta}], d \in [\delta,1]} \bigg\{ Azy - (1-d)k\lambda_e\theta_e + \beta\Big[\xi\mathbb{E}_0[W_U(z',h',\hat{A})]\\
&+ (1-\xi)\bigg(d\mathbb{E}_E[W_U(z',h',\hat{A})] + (1-d)\bigg(\int_Y \max_{c_e \in [0,1]} \big\{\lambda_e p(\theta_e)c_e\mathbb{E}_E[W_E(z',h',y',\hat{A})]\\
&\qquad + (1-\lambda_e p(\theta_e)c_e)\mathbb{E}_E[W_E(z',h',y,\hat{A})]\big\}f(y')dy'\bigg)\bigg)\Big]\bigg\}
\end{aligned}$$

where $\overline{\theta} = \max\{\overline{\theta}_e, \overline{\theta}_u\}$. Since (6) aggregates W_U and W_E for all $(z,h) \in Z \times H$ and $y \in Y$, and u and e are predetermined, the solution to (6) is unique.

(iii) This section proves the monotonicity of the value functions (7) and (8). Define the space of bounded, continuous functions that are nondecreasing in each of their arguments

as $B(\Psi)$, and the space of bounded, continuous functions that are strictly increasing in h as $B'(\Psi) \subset B(\Psi)$. Define the operator T as

$$(Tf_U)(z,h,A) = \max_{\theta_u \in [0,\bar{\theta}]} \Big\{ h - k\theta_u + \beta \Big[\xi \mathbb{E}_0[f_U(z',h',\hat{A})]$$
$$+(1-\xi)\Big(\int_Y \max_{c_u \in [0,1]} \big\{ p(\theta_u)c_u \mathbb{E}_U[f_E(z',h',y,\hat{A})] + (1-p(\theta_u)c_u)\mathbb{E}_U[f_U(z',h',\hat{A})] \big\} f(y)dy \Big) \Big] \Big\}$$

where f_U and f_E are nondecreasing, bounded, and continuous functions in each of their arguments. Suppose $\theta_u^* \equiv \theta_u^*(z,h,A)$ achieves the maximum of the above equation, and take any $\tilde{h} > h$. Then

$$(Tf_U)(z,h,A) = h - k\theta_u^* + \beta \Big[\xi \mathbb{E}_0[f_U(z',h',\hat{A})]$$
$$+(1-\xi)\Big(\int_Y \max_{c_u \in [0,1]} \big\{ p(\theta_u^*)c_u \mathbb{E}_U[f_E(z',h',y,\hat{A})] + (1-p(\theta_u^*)c_u)\mathbb{E}_U[f_U(z',h',\hat{A})] \big\} f(y)dy \Big) \Big]$$

$$\begin{aligned}
&< \tilde{h} - k\theta_u^* + \beta \Big[\xi \mathbb{E}_0[f_U(z',h',\hat{A})] \\
&+ (1-\xi)\Big(\int_Y \max_{c_u \in [0,1]} \big\{ p(\theta_u^*)c_u \mathbb{E}_U[f_E(z',h',y,\hat{A})] + (1-p(\theta_u^*)c_u)\mathbb{E}_U[f_U(z',h',\hat{A})] \big\} f(y)dy \Big) \Big] \\
&\leq \max_{\theta_u \in [0,\bar{\theta}]} \Big\{ \tilde{h} - k\theta_u + \beta \Big[\xi \mathbb{E}_0[f_U(z',\tilde{h}',\hat{A})] \\
&+ (1-\xi)\Big(\int_Y \max_{c_u \in [0,1]} \big\{ p(\theta_u)c_u \mathbb{E}_U[f_E(z',\tilde{h}',y,\hat{A})] + (1-p(\theta_u)c_u)\mathbb{E}_U[f_U(z',\tilde{h}',\hat{A})] \big\} f(y)dy \Big) \Big] \Big\} \\
&= (Tf_U)(z,\tilde{h},x)
\end{aligned}$$

Where, with a slight abuse of notation, $\mathbb{E}_U[f_U(z',\tilde{h}',\hat{A})] \equiv \mathbb{E}_U[f_U(z',h',\hat{A})|z,\tilde{h},A]$. The inequality on the second line follows from the fact that Q_U and Q_E are monotone transition functions by Assumption 1. Note that the choice of c_u^* is independent of θ_u^* as it is chosen after meeting a firm and drawing match-specific productivity. Since $B'(\Psi)$ is a closed subset of $B(\Psi)$ and $T(B'(\Psi)) \subset B'(\Psi)$, it follows from SLP Theorem 4.7 that $W_U \subset B'(\Psi)$. Similarly, it can be shown that $W_U(z,h,A)$ is weakly increasing in z and A, and $W_E(z,h,y,A)$ is strictly increasing in z, y, and A and weakly increasing in h.

(iv) From part (ii), the policy correspondences $\theta_u^*(z,h,\psi)$, $\theta_e^*(z,h,y,\psi)$, $c_u^*(z,h,y,\psi)$, $c_e^*(z,h,y,y',\psi)$ and $d(z,h,y,\psi)$ solve (7) and (8). Since neither the expression to be maximized nor the constraint depends on (u,e), $\theta_u^*(z,h,\psi)$, $\theta_e^*(z,h,y,\psi)$, $c_u^*(z,h,y,\psi)$, $c_e^*(z,h,y,y',\psi)$ and $d(z,h,y,\psi)$ depend on ψ only through A and not on (u,e).

B.2 Proof of Lemma 1

The expectation operators in the statement of Lemma 1 are given by the following expressions.

$$\mathbb{E}_i(V(\alpha, s', y, \hat{\psi})) = \int_S \int_\Psi V(\alpha, s', y, \hat{\psi})\ d\Gamma(\psi, d\hat{\psi})\ dQ_i(s, ds'), \quad i = U, E$$

$$\mathbb{E}_0(V(0, s', y, \hat{\psi})) = \int_S \int_\Psi V(0, s', y, \hat{\psi})\ d\Gamma(\psi, d\hat{\psi})\ f_0(s')ds'$$

for $s \in S \equiv Z \times H$ and where Γ is the perceived law of motion for the aggregate state ψ, which must be consistent with the actual law of motion in any equilibrium.

($\Rightarrow$) Suppose an equilibrium exists. Then $(V_U, V_M, x_u, r_u, x_e, r_e, d)$ satisfy (A1), (A4), and (A5). If $\theta_u = 0$, x_u is not pinned down by the free entry condition, but the probability that a worker meets a vacancy in that submarket is 0, therefore let $x_u = 0$ when $\theta_u = 0$, and similarly let $x_e = 0$ when $\theta_e = 0$.[3]

If $\theta_i > 0$, solving (A5) for x_i,

$$x_i = \frac{1}{\overline{F}(r_i)} \int_{r_i}^{\overline{y}} \mathbb{E}_i(V_M(z', h', y', \hat{\psi}))f(y')dy' - \frac{k}{\overline{F}(r_i)\beta(1-\xi)q(\theta_i)}, \quad i = u, e$$

Plugging in for x_i from the free entry condition and noting that $\frac{p(\theta)}{q(\theta)} = \theta$, the combined value function for all agents in this equilibrium can be written

$$\begin{aligned} V(\alpha, z, h, y, \psi) = \alpha\Bigg(& Azy - k\lambda_e(1-d^*)\theta_e^* + \beta(1-\xi)\big[d^*\mathbb{E}_E V(0, z', h', y, \hat{\psi}) \\ & +(1-d^*)\big(\mathbb{E}_E V(1, z', h', y, \hat{\psi}) + \lambda_e p(\theta_e^*) \int_{r_e^*}^{\overline{y}} \mathbb{E}_E(V(1, z', h', y', \hat{\psi}) - V(1, z', h', y, \hat{\psi}))f(y')\ dy'\big)\big]\Bigg) \\ & + (1-\alpha)\Bigg(h - k\theta_u^* + \beta(1-\xi)\big(\mathbb{E}_U(V(0, z', h', y, \hat{\psi})) \\ & + p(\theta^*) \int_{r_u^*}^{\overline{y}} \mathbb{E}_U(V(1, z', h', y', \hat{\psi}) - V(0, z', h', y, \hat{\psi}))f(y')\ dy'\big)\Bigg) + \beta\xi\mathbb{E}_0(V(0, z', h', y, \hat{\psi})) \end{aligned} \tag{A7}$$

Since (x_u^*, r_u^*) maximize V_U and (x_e^*, r_e^*, d^*) maximize V_M, θ_u^* and θ_e^* must maximize the above expression, giving us (A6). Since θ_i is bounded below by 0, it must be shown that $\exists\ \overline{\theta_i} < \infty$ such that $\theta_i \in [0, \overline{\theta_i}]\ i = u, e\ \forall\ s, \psi$. By definition of the probability q, when $\theta \to \infty$, $q(\theta) \to 0$. Thus for large enough θ, the (binding) free entry condition is violated. It follows that $\exists\ \overline{\theta_i} < \infty$ for $i = u, e$.

In addition, $\beta < 1$ and the per-period payoff $\alpha(Azy - k\lambda_e\theta_e(1-d)) + (1-\alpha)(h - k\theta_u)$ is continuous and bounded. Therefore the equilibrium solves (A6).

[3] If this were not the case, then there would exist some inactive submarkets with a positive wage in which no matches would occur.

($\Leftarrow$) Take any solution to (A6). For $\alpha = 0$,

$$V(0,z,h,y,\psi) = \max_{\theta\in[0,\overline{\theta}]} \big\{h - k\theta + \beta(1-\xi)\big(\mathbb{E}_U(V(0,z',h',y,\hat{\psi}))$$

$$+\int_Y \max_{c\in[0,1]} \{p(\theta)c\mathbb{E}_U(V(1,z',h',y'\hat{\psi}) - V(0,z',h',\hat{\psi}))'\} f(y')dy\big)\big\} + \beta\xi\mathbb{E}_0(V(0,z',h',y,\hat{\psi}))$$

if $\overline{F}(r)x_u = \int_{r_u}^{\overline{y}} \mathbb{E}_U(V_M(z',h',y',\hat{\psi}))f(y')\ dy' - \frac{k}{\beta(1-\xi)q(\theta)}$, then

$$V_U(z,h,\psi) = \sup_x \Big\{h + \beta(1-\xi)\Big[(1 - p(\theta(x,r,z,h,\psi))\overline{F}(r))\mathbb{E}_U(V_U(z',h',\hat{\psi}))$$
$$+ p(\theta(x,r,z,h,\psi))\overline{F}(r)x_u\Big] + \beta\xi\mathbb{E}_0(V_U(z',h',\hat{\psi}))\Big\}$$

Which satisfy (A1) and (A5) for $\theta > 0$. Finally, for $\theta_u = 0$, $p(0) = 0$, and by assumption $x_u = 0$, therefore $V(0,z,h,y,\psi)$ can be written as

$$V_U(z,h,\psi) = h + \beta\big((1-\xi)\mathbb{E}_U(V_U(z',h',\hat{\psi})) + \xi\mathbb{E}_0(V_U(z',h',\hat{\psi}))\big)$$

which is equivalent to $V_U(z,h,\psi)$ when $x_u = 0$. Similarly, it can be shown that the value function evaluated at $\alpha = 1$ satisfies (A4). Thus any solution to (A6) is an equilibrium.

B.3 Proof of Theorem 2

Let $(\theta_u, \theta_e, V_U, V_M, x_u, r_u, x_e, r_e, d)$ be an equilibrium and let $V : [0,1] \times Z \times H \times Y \times \Psi$ be defined as

$$V(0,z,h,y,\psi) = V_U(z,h,\psi)\ \forall(z,h,y,\psi) \in Z \times H \times Y \times \Psi$$

$$V(1,z,h,y,\psi) = V_M(z,h,y,\psi)\ \forall(z,h,y,\psi) \in Z \times H \times Y \times \Psi$$

I first show the existence and uniqueness of the solution to (A6). It is clear that the sets of feasible values for θ and d are non-empty, compact, and continuous. The period utility function is bounded and continuous, $\xi \in [0,1]$ and $\beta \in (0,1)$. It immediately follows that a solution to (A6) exists.

By the concavity of firms' expected profit function and of the composite function $p'(q^{-1}(\cdot))$, there is a unique choice of x for each (z,h,y,ψ). Let $\Omega = [0,1] \times Z \times H \times Y \times \Psi$ and let $C(\Omega)$ be the space of continuous bounded functions $R : \omega \to \mathbb{R}$ for $\omega \in \Omega$ with the sup norm. Let $T : C(\Omega) \to C(\Omega)$ denote the operator associated with (A6). It is easy to establish that Blackwell's sufficient conditions for a contraction are satisfied by T, thus by SLP Theorem 4.6 the mapping $TR = R$ admits a unique solution. Finally, note that since all choice and state variables are bounded, $V(\alpha,z,h,y,\psi)$ is bounded and by Theorem 4.3, the unique solution to $TR = R$ coincides with the solution to (A6), thus the equilibrium is unique.

I now prove independence of the value and policy functions from (u,e). Following

Menzio and Shi (2011), consider an arbitrary function $R \in C'(\Omega)$, denoting the set of continuous, bounded functions mapping $Z \times H \times Y \times \mathbf{A} \to \mathbb{R}$. It is straightforward to show that $T : C'(\Omega) \to C'(\Omega)$, thus TR depends on ψ only through A. Therefore $V(\alpha, z, h, y, \psi) = V(\alpha, z, h, y, A)$ is the unique fixed point of (A6). Since q is strictly decreasing and convex, $\theta(x, r, z, h, \psi)$ is uniquely pinned down by (A5) and therefore only depends on ψ through A.

Finally, replacing ψ with A in (A1) and (A4) proves the independence of both value functions from (u, e). It follows that the policy functions $x_u(z, h, \psi), r_u(z, h, \psi), x_e(z, h, y, \psi)$, $r_e(z, h, y, \psi)$, and $d(z, h, y, \psi)$ depend on ψ only through A.

B.4 Proof of Proposition 1

This proof shows that the equilibrium allocation and efficient allocation coincide, that is, $\theta(x_e(s, y, A), r_e(s, y, A), s, A) = \theta_e^*(s, y, A)$, $\theta(x_u(s, A), r_u(s, A), s, A) = \theta_u^*(s, A)$, $r_e(s, y, A) = r_e^*(s, y, A)$, $r_u(s, A) = r_u^*(s, A)$, and $d(s, y, A) = d^*(s, y, A)$, where $s \in Z \times H$. Define $W'(0, z, h, y, A) = W_U(z, h, A)$ and $W'(1, z, h, y, A) = W_E(z, h, y, A)$ where $W_U(z, h, A)$ solves (7) and $W_E(z, h, y, A)$ solves (8). One can write the combined value function of the planner as

$$W'(\alpha, z, h, y, A) = \alpha\Bigg(\max_{\theta\in[0,\overline{\theta}], d\in[\delta,1]} \Bigg\{ Azy - k\lambda_e\theta(1-d) + \beta(1-\xi)\big[d\mathbb{E}_E W'(0, z, h, y, \psi)$$

$$+(1-d)\big(\mathbb{E}_E W'(1, z', h', y, \psi) + \int_Y \max_{c\in[0,1]} \big\{\lambda_e p(\theta) c \mathbb{E}_E\big(W'(1, z', h', y', \psi) - W'(1, z', h', y, \psi)\big)\big\} f(y')\, dy'\big)\big]\Bigg\}\Bigg)$$

$$+ (1-\alpha) \max_{\theta\in[0,\overline{\theta}]} \big\{h - k\theta + \beta(1-\xi)\big(\mathbb{E}_U(W'(0, z', h', y, \hat{\psi}))$$

$$+ \int_Y \max_{c\in[0,1]} \{p(\theta) c \mathbb{E}_U(W'(1, z', h', y'\hat{\psi}) - W'(0, z', h', y, \hat{\psi}))'\} f(y') dy\big)\big\} + \beta\xi\mathbb{E}_0(W'(0, z', h', y, \hat{\psi})) \tag{A8}$$

From (A8) it is clear that $W'(\alpha, z, h, y, A)$ satisfies (A6). Since V is unique, it must be the case that $V_U(z, h, A) = W_U(z, h, A)$ and $V_M(z, h, y, A) = W_E(z, h, y, A)$.

By definition, the allocation that solves (A8) is the solution to the planner's problem, $(\theta_u^*(z, h, A), r_u^*(z, h, A), \theta_e^*(z, h, y, A), r_e^*(z, h, y, A), d^*(z, h, y, A))$, and the allocation that solves (A6) corresponds to the decentralized equilibrium, $(\theta_u(z, h, A), r_u(z, h, A), \theta_e(z, h, y, A)$, $r_e(z, h, y, A), d(z, h, y, A))$.

C Estimation: Details

To obtain the estimate of the job-finding probability as a function of tenure in the previous job, I identify workers who transitioned from unemployment to employment over two consecutive months of CPS interviews, and who also participated in the Displaced Worker Survey (DWS) the month they reported being unemployed between 1994 and 2010. For the sample period considered, 1994-2010, the DWS was administered 9 times. This sample includes 5,634 workers. Of these, 898 (16%) transitioned to employment in the second month.

In order to identify the depreciation of home skills during employment, I consider only those workers with unemployment spells shorter than one month at the time of the DWS. In the model, these short-duration workers are those whose productivities most resemble their productivities at the time of separation. I regress a dummy variable equal to 1 if the worker transitioned from U to E and 0 if they reported being unemployed in both months on months of tenure in the previous job reported in the DWS for workers with up to 15 years of tenure, controlling for observables.

Transition rates are computed using the monthly employment status of all prime age individuals in the 1996 through 2008 SIPP panels. See Section D.2 for further details on the SIPP sample.

C.1 Robustness of the Quantitative Model

This section presents results for the steady state model under several different parameter choices in order to evaluate the robustness of the model's quantitative predictions. There are two exogenously-set parameters which may alter the predictions of the model, namely, the probability of death ξ and the elasticity of the matching function with respect to vacancies, γ. The first robustness check is to increase the probability of death, corresponding to an expected lifetime of 30 years, rather than the 40 year expected lifetime assumed in the benchmark. The final two robustness checks vary the elasticity of the matching function, first decreasing its value to match the value estimated in the CPS from 1951-2003 by Shimer (2005) of .28, and then increasing its value to .5. Each of these robustness checks leaves all other parameters identical to those in Table 3, and reports mean values and 95% confidence intervals from simulations of 20,000 workers over 500 months, discarding the first 50 months.

C.2 Changes in the Death Probability

Tables C.1 and C.2 show how the model-implied moments change with an increase in the probability of death from $\xi = .0021$ to $\xi = .0027$. This decreases the effective discount factor, making agents more impatient since future payoffs are less likely to be realized. The tables show that this 25% decrease in the average working lifetime has only a small effect on the targeted parameters. In particular, the shorter lifetime somewhat increases the UE rate and decreases the relative value of non-market work. The reemployment wage and job-finding probability fall by slightly less with duration as fewer workers remain unemployed for long durations, reflected in a slightly higher participation rate.

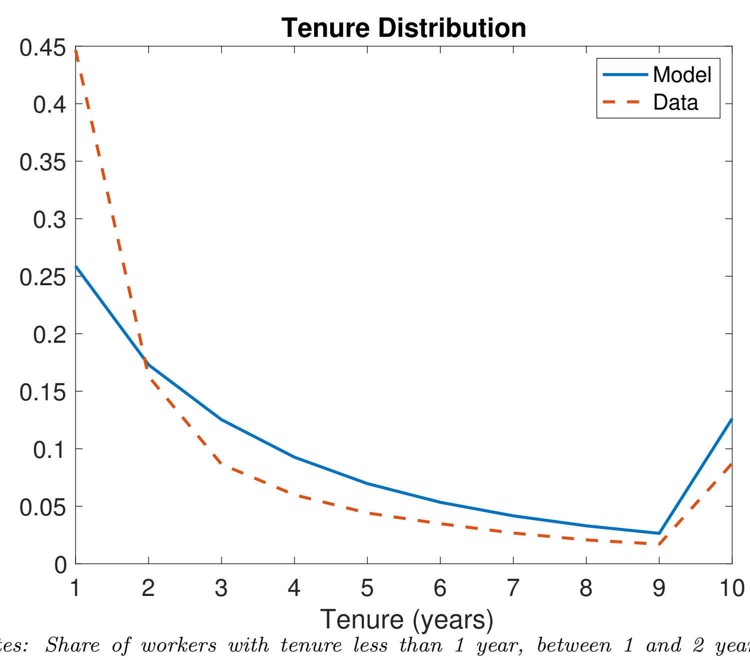
Table C.1: Targeted Moments: Higher Death Probability

Description	Target	Model [95% CI]	Change from Benchmark
Annual interest rate	5%	5%	0%
Average working lifetime	40 years	30 years	-25%
Matching function elasticity w.r.t v	.4	.4	0%
NE Rate (Quarterly)	.053	.046 [.041, .051]	0 ppts
EE Rate (Monthly)	.014	.015 [.014,.016]	+0.1 ppts
UE Rate (Quarterly)	.228	.235 [.213, .255]	+0.7 ppts
EU Rate (Quarterly)	.016	.016 [.015, .017]	-0.1 ppts
UN Rate (Quarterly)	.192	.201 [.173, .225]	-0.4 ppts
Relative value of non-market work	.71	.69 [.663, .728]	-2.8%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	0.73% [-2.4%, 2.5%]	0 ppts
Average Skill Depreciation (Annual)	.20	.12 [0, .55]	0 ppts
Tenure Distribution	See Figure A1		

Table C.2: Untargeted Moments: Higher Death Probability

Description	Data	Model [95% CI]	Change from Benchmark
% change, log reemployment wage	−0.5%	−8.5%	+0.4 ppts
% change, job-finding probability	−53.8%	−71.6%	+1.8 ppts
Unemployment rate	4.2%	3.6%	-0.2 ppts
Labor force participation rate	84%	82%	+1 ppt
Initial job-finding probability (1 month)	.145	.292	-0.9 ppts

Figure A1: Tenure Distribution, High Death Probability



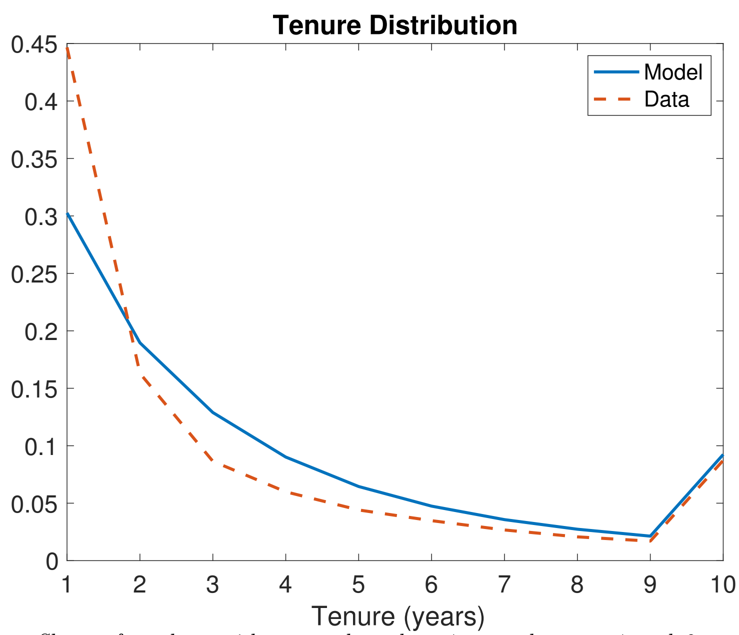
Notes: Share of workers with tenure less than 1 year, between 1 and 2 years, ..., greater than 10 years. Dashed line shows the empirical tenure distribution for workers between 25 and 54 in the SIPP, solid line shows the distribution computed from model simulations.

C.3 Changes in the Matching Function Elasticity

This section explores how changes in the elasticity of the matching function with respect to vacancies affect the results of the quantitative model. Two possibilities are explored relative to the benchmark value of 0.4. First, Shimer (2005) estimates an elasticity of 0.28 using CPS data from 1951-2003. Second, a higher value of .5 is used. Tables C.3 and C.4 show the targeted and untargeted moments of the model using the low elasticity of the matching function, and Tables C.5 and C.6 are the corresponding tables for the high elasticity.

A decrease in the elasticity of the matching function by 30% results in a large increase in the UE rate, a decline in the UN rate, a smaller decline in the job-finding rate and a larger decline in the wage with duration. Differently, an increase in the elasticity by 25% leads to a smaller increase in the UE rate, a larger decline in the UN rate, and a larger decline in the job-finding rate.

Figure A2: Tenure Distribution, 0.28 Matching Function Elasticity



Notes: Share of workers with tenure less than 1 year, between 1 and 2 years, ..., greater than 10 years. Dashed line shows the empirical tenure distribution for workers between 25 and 54 in the SIPP, solid line shows the distribution computed from model simulations.

The cyclical patterns of all of the robustness checks discussed in this section are largely identical to those of the baseline model. Specifically, the small but highly persistent decline in labor force participation, the large asymmetric response of the unemployment rate, the fall in the job-finding probability, and the pro-cyclical behavior of the average home productivity of the unemployed. For brevity, figures similar to Figure 4 with the alternative parameters discussed in this section are omitted.

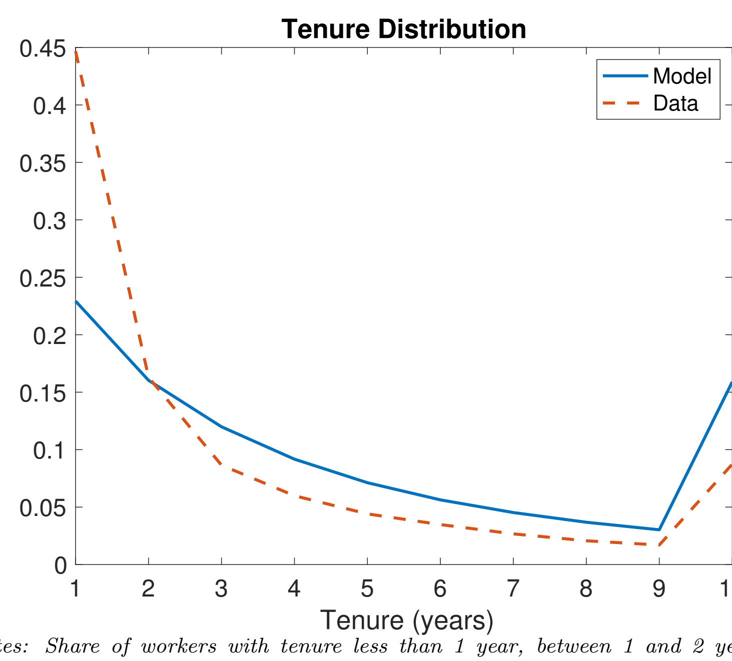
Table C.3: Targeted Moments: 0.28 Matching Function Elasticity

Description	Target	Model [95% CI]	Change from Benchmark
Annual interest rate	5%	5%	0%
Average working lifetime	40 years	40 years	0%
Matching function elasticity w.r.t v	.28	.28	-30%
NE Rate (Quarterly)	.053	.045 [.040, .048]	-0.1 ppts
EE Rate (Monthly)	.014	.020 [.019,.022]	+0.6 ppts
UE Rate (Quarterly)	.228	.266 [.245, .287]	+3.8 ppts
EU Rate (Quarterly)	.016	.018 [.016, .022]	+0.1 ppts
UN Rate (Quarterly)	.192	.187 [.117, .220]	-1.8 ppts
Relative value of non-market work	.71	.71 [.677, .761]	0%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	0.36% [-2.1%, 2.5%]	-0.4 ppts
Average Skill Depreciation (Annual)	.20	.11 [0, .49]	-1.0 ppts
Tenure Distribution	See Figure A2		

Table C.4: Untargeted Moments: 0.28 Matching Function Elasticity

Description	Data	Model	Change from Benchmark
% change, log reemployment wage	−0.5%	−16.5%	-7.6 ppts
% change, job-finding probability	−53.8%	−70.9%	+2.5 ppts
Unemployment rate	4.2%	3.6%	-0.2 ppts
Labor force participation rate	84%	82%	+1.0 ppts
Initial job-finding probability (1 month)	.145	.340	+3.9 ppts

Figure A3: Tenure Distribution, 0.5 Matching Function Elasticity



Notes: Share of workers with tenure less than 1 year, between 1 and 2 years, ..., greater than 10 years. Dashed line shows the empirical tenure distribution for workers between 25 and 54 in the SIPP, solid line shows the distribution computed from model simulations.

Table C.5: Targeted Moments: Matching Function Elasticity 0.5

Description	Target	Model [95% CI]	% Change from Benchmark
Annual interest rate	5%	5%	0%
Average working lifetime	40 years	40 years	0%
Matching function elasticity w.r.t v	.5	.5	+25%
NE Rate (Quarterly)	.053	.031 [.028, .034]	-1.5 ppts
EE Rate (Monthly)	.014	.011 [.010,.013]	-0.3 ppts
UE Rate (Quarterly)	.228	.230 [.210, .256]	+0.2 ppts
EU Rate (Quarterly)	.016	.014 [.013, .015]	-0.3 ppts
UN Rate (Quarterly)	.192	.172 [.153, .191]	-3.3 ppts
Relative value of non-market work	.71	.67 [.636, .704]	-5.6%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	1.24% [-2.4%, 2.4%]	+0.5 ppts
Average Skill Depreciation (Annual)	.20	.13 [0, .55]	+1.0 ppts
Tenure Distribution	See Figure A3		

Table C.6: Untargeted Moments: Matching Function Elasticity 0.5

Description	Data	Model	Change from Benchmark
% change, log reemployment wage	−0.5%	−9.7%	-0.8 ppts
% change, job-finding probability	−53.8%	−78.4%	-5.0 ppts
Unemployment rate	4.2%	3.5%	-0.3 ppts
Labor force participation rate	84%	78%	-3.0 ppts
Initial job-finding probability (1 month)	.145	.305	+0.4 ppts

D Data

This section summarizes the evidence pointing to the effect of the unemployed's outside option on duration. Then, it summarizes the method and regression estimates used to construct Figure 2 and performs several robustness checks.

D.1 Measuring the Effect of the Outside Option on Duration

Unemployment duration can plausibly be affected by three factors: the "luck" of the worker in finding a job, her market-related productivity, and her outside option. In this section I first consider the components of the outside option and argue that it is infeasible to disentangle each factor using available data sources. Then, I present novel results relating to the causal effect of changes in the non-pecuniary aspects of the outside option on duration.

D.1.1 Evidence for the Model Mechanism

If the outside option changes with duration, one expects to see individual workers adjust their allocations of time spent in different activities over the course of an unemployment spell. In order to disentangle changes in the components of the outside option, it would be necessary to measure the subjective value of these activities as a time series for individual workers. Measuring the time allocated to home production as a function of unemployment duration in a pooled cross section, as is possible for instance using the American Time Use Survey (ATUS), cannot tell us whether workers enjoy more or less utility from the activity with duration. I therefore will not attempt to analyze the individual components of the outside option and instead will consider the activities comprising the outside option – that is, leisure and home work – together.

To provide evidence in support of a key assumption of the model below, I show that changes in the allocation of time to activities comprising the outside option cause changes in unemployment duration using an instrumental variables approach. The reason for using an exogenous shock to the outside option is that the causal relationship between the outside option and unemployment duration could run in both directions. On the one hand, workers with longer durations may be discouraged, causing them to allocate more time to leisure or home work rather than searching for jobs. On the other hand, workers with a higher outside option may be able to better enjoy the benefits from home production or leisure, for example by having more time to improve one's piano playing, causing them to reject more job offers that would decrease the time available to perform these activities.

I proceed in two steps. In the first stage, using data from the ATUS and the National Centers for Environmental Information (NCEI)[4] I show that changes in monthly rainfall in a given state significantly affect unemployed workers' time use in activities related to the outside option. Then, using these exogenous rainfall shocks as an instrument for

[4]This data set is known as nCLIMDIV. For details, see https://www.ncdc.noaa.gov/temp-and-precip/national-temperature-index/background/climdiv .

changes in time use, I show that an increase in time dedicated to the outside option causes unemployment duration to increase.

The ATUS respondents are a subset of recent CPS interviewees from 2003 onward. The sample is restricted to unemployed individuals, between the ages of 23 and 55, with durations up to 26 weeks to reduce the potential effect of unemployment benefit expiration on time use. Further, I consider only individuals whose spouse or partner is employed at the time of the survey in order to study those with access to intra-household insurance coming from a working partner.[5]

Using the method of Krueger and Mueller (2016), I impute duration for individuals who are unemployed in the CPS and the ATUS as the duration reported in the CPS plus the length of time in weeks between the CPS and ATUS interviews. For those individuals employed in the CPS and unemployed in the ATUS, duration is set to half the time in weeks between the two interviews. Finally, duration for individuals who are employed in the ATUS is set to 0. This ensures that workers who are out of the labor force have recorded duration only if they reported being unemployed in the CPS and recently transitioned out of the labor force.

The precipitation data contains the monthly total rainfall in inches by state since 1895, defined as $rain$. The instrument for the outside option I will consider here, $rain\_dev$, is defined as the deviation from the monthly average rainfall for each state over the sample, normalized by the monthly standard deviation. More precisely, denoting the state as s and the month as t,

$$rain\_dev_{st} = \frac{rain_{st} - \overline{rain}_s}{\sigma(rain_s)}$$

where $\overline{rain}_s$ is the average rainfall in a given month.

Time use categories are constructed following the classification in the Online Appendix of Aguiar et al. (2013). The analysis in this paper uses the multi-year micro-data files from 2003 to 2015 available from the BLS. Leisure time is defined as time spent sleeping, watching television, socializing, personal care, and "other leisure" (playing games, with pets, attending museums). Non-market time is comprised of "core" home production (cooking, cleaning, financial management), home ownership (interior and exterior home maintenance), purchasing goods and services, and others' care.

The relative value of activities such as leisure and non-market work is affected by the range of activities one can do, which in turn is affected by the weather. Table D.7 shows the first and second stage estimates from the 2SLS regressions using the instrument discussed above. All regressions include time and state fixed effects and a set of individual characteristics. Standard errors are clustered at the regional level (Northeast, Midwest, South, and West).

The regression estimates for the first stage are presented in the first column. The dependent variable is the total time use in leisure and non-market activities, the major

[5] The model abstracts from the fact that one generally must use purchased inputs to produce consumption of the outside option. For example, in order to enjoy leisure, one must pay for a television, which cannot be done through home production alone. Therefore the model implicitly assumes intra-household insurance like that of the individuals considered in this section.

Table D.7: ATUS Estimates: Time Use in Outside Option and Duration

	First Stage: Leisure+Non-market	Second Stage: Duration
rain_dev	-88.2**	
	(37.1)	
Leisure+ Non-Market		3.21e-03*
		(1.66e-03)
Time and State FE	X	X
Ind. Characteristics	X	X
R^2	.205	
N	1,770	1,770

Notes: ATUS: January 2003-December 2015, monthly; duration reported in weeks. Universe: workers unemployed at the time of the interview with reported duration up to 26 weeks, ages 23-55, whose spouse or partner is employed. Time and state fixed effects include the ATUS interview month, year and month of the final CPS interview, and state. Individual characteristics include gender, race, education, marital status, and quadratic terms for age and number of children. Daily minutes are multiplied by 7 to get the weekly estimate. Observations are weighted by their ATUS final weight. ***, **, and * indicate significance at the 1%, 5%, and 10% levels. Standard errors are clustered at the regional level (Northeast, Midwest, South, and West).

components of the outside option. The estimate suggests that a positive rainfall shock negatively affects time allocated to the overall outside option. The result in the second column shows that devoting more time to the outside option causes unemployment duration to increase.

D.1.2 Details and Robustness

Table D.8 reproduces the first stage regression shown in Table D.7 in detail. Year, month, state, and education fixed effects are omitted for brevity. Table D.9 shows the 2 Stage Least Squares estimates excluding time spent sleeping from the endogenous time use variable used in Section D.1. Finally, Figure A4 shows a histogram of the instrument $rain\_dev_{st}$ used in Section D.1, overlay with a normal density.

Table D.8: ATUS Estimates: Time Use in Outside Option

	First Stage: Leisure+Non-market
rain_dev	-88.2**
	(37.1)
Male	39.9
	(49.8)
Age	-29.7
	(35.1)
Age^2	0.48
	(0.38)
# children	-450.8***
	(85.5)
# children^2	70.2***
	(17.3)
White	157.9
	(129.1)
Black	-138.2*
	(75.0)
Asian	-126.2
	(121.5)
R^2	.192
N	1,654

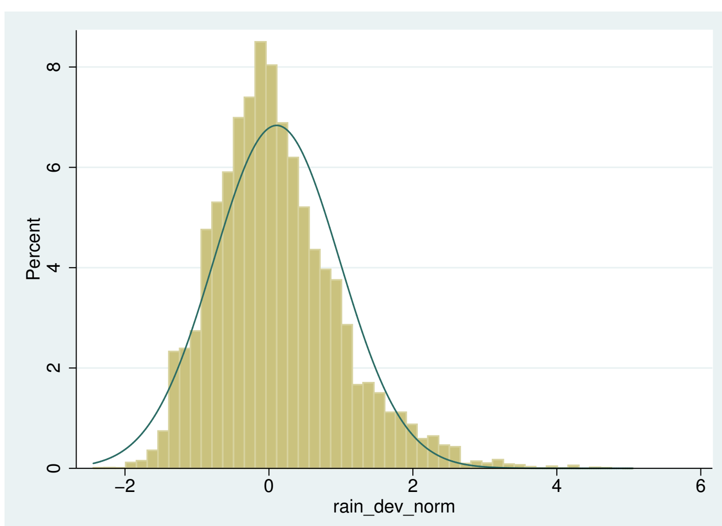
Notes: ATUS: January 2003-December 2015, monthly; duration reported in weeks. Universe: workers unemployed at the time of the interview with reported duration up to 26 weeks, ages 23-55, whose spouse or partner is employed. Time and state fixed effects include the ATUS interview month, year and month of the final CPS interview, and state. Individual characteristics include gender, race, education, marital status, and quadratic terms for age and number of children. Daily minutes are multiplied by 7 to get the weekly estimate. Observations are weighted by their ATUS final weight. ***, **, and * indicate significance at the 1%, 5%, and 10% levels. Standard errors are clustered at the regional level (Northeast, Midwest, South, and West).

Table D.9: ATUS Estimates: Time Use in Outside Option and Duration, Excluding Time Asleep

	First Stage: Leisure (excl. sleep) +Non-market	Second Stage: Duration
$rain\_dev_{st}$	-135.7**	
	(53.4)	
Leisure (excl. sleep)+ Non-Market		2.09e-03
		(1.49e-03)
Time and State FE	X	X
Ind. Characteristics	X	X
R^2	.222	
N	1,770	1,770

Notes: ATUS: January 2003-December 2015, monthly; duration reported in weeks. Universe: workers unemployed at the time of the interview with reported duration up to 26 weeks, ages 23-55, whose spouse or partner is employed. Time and state fixed effects include the ATUS interview month, year and month of the final CPS interview, and state. Individual characteristics include gender, race, education, marital status, and quadratic terms for age and number of children. Daily minutes are multiplied by 7 to get the weekly estimate. Observations are weighted by their ATUS final weight. ***, **, and * indicate significance at the 1%, 5%, and 10% levels. Standard errors are clustered at the regional level (Northeast, Midwest, South, and West).

Figure A4: Distribution of $rain\_dev_{st}$



Notes: Distribution of normalized rainfall instrument used in Section D.1. Data is monthly by state, 1994-2015.

D.2 Details on the SIPP Sample

I merge the monthly data files for the SIPP using the sample unit identifier and person number to identify individuals and append the panels together to create the final data set. The choice of time period is guided by the availability and comparability of SIPP panels: the survey underwent a substantial redesign in 1996 to improve the quality of the longitudinal estimates, and the last full panel available to date is the 2008 panel, which covers the period through the end of 2013. I keep only individuals who appear in at least two waves of the survey and define a worker's employment status in the second week of each month, following Fujita and Moscarini (2017), to stay as close as possible to the CPS definition of employment status. To account for classification error of employment status, spells are "denunified" following Elsby et al. (2015). To do so, I recode spells of UNU to UUU and NUN as NNN to avoid overstating the transition rate between unemployment and out of the labor force.

To measure the full unemployment spell, I use workers who report being employed in at least one month prior to the unemployment spell. Unemployment duration is computed as the sum of the number of weeks looking if a worker reports being unemployed in a given month. To compute duration across months, I take the sum of duration in two consecutive months for workers who were unemployed in the last week of the previous month and the first week of the current month. I use monthly earnings recorded before deductions, that are not imputed and deflate them using CPI for all urban consumers. Finally, I only consider workers who are not in education, not self-employed, and not retired.

D.3 Details: Figure 2

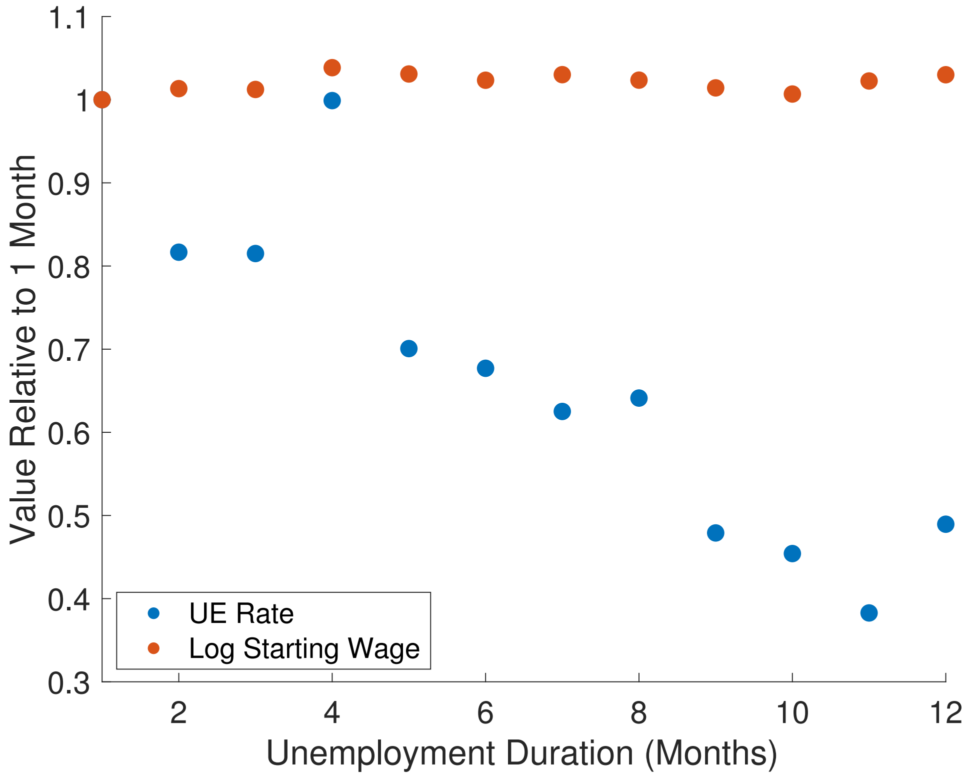
In the SIPP, I identify those workers who report being unemployed in month t and being employed full-time in month $t + 1$. Figure A5 shows the change in the raw data with duration, normalizing the starting wage and month-to-month job-finding rate to one at one month of duration. As can be seen from the figure, there is a stark difference between the change in the job-finding rate and starting wage as a function of duration.

To show that this pattern is also present after controlling for observables, Figure 2 is drawn using regression estimates for equations (1) and (2) in the main text, holding all other covariates at their means. The figure shows the strong negative effect of duration on the probability of transitioning from unemployment to employment, but a much weaker effect on the reemployment wages of those workers who transition.

D.4 Robustness: SIPP

This section checks the robustness of the benchmark results shown in Tables 1 and 2. Table D.10 shows that the sample on which I perform the fixed effects regression in Table 1 is highly similar in terms of observable characteristics to the pool of unemployed who do not experience multiple spells. Note that in order to have multiple starting wages, an individual must have two months in which UE=1, whereas in order to have multiple UE observations, an individual needs not satisfy this constraint. For comparability between

Figure A5: Relative UE Rate and Wage, Raw Data



Notes: Average UE rate and log wage for all workers ages 25-54 transitioning from unemployment into full-time employment in the SIPP sample, 1/1996-11/2013.

the results of the two fixed effects regressions, I restrict both specifications to the sample with at least two complete UE spells. For additional robustness checks, Tables D.11 and D.12 show identical estimates to those in column 1 of Tables 1 and 2 with additional sample selection criteria or regressors. Column 1 in each table shows the results allowing for left-censored spells, that is, unemployment spells that are in progress when an individual enters the sample. Column 2 shows the results for regressions only for workers who were not recalled to their previous job.[6] Column 3 shows results for the sub-sample of men, and column 4 for women.

[6] Following Fujita and Moscarini (2017) I identify recalls in the data as those workers whose across-wave employer number is the same pre- and post-unemployment spell. As they discuss, one must use longitudinal weights to account for attrition in order to avoid underestimating the level of recalls. Including these weights do not significantly affect the results shown here.

Table D.10: Average Observable Characteristics, Full Sample vs Fixed Effects Sample

	Full Sample	> 1 UE Spells
Age	38.8	38.2
Share Male	.56	.65
Share Married	.47	.45
Share HS Graduate	.31	.28
Share Some College	.34	.38
Share College +	.18	.15
Share Collecting UI	.40	.41
Weeks Unemployed	16.6	14.6

Notes: SIPP Sample, 1/1996-11/2013, monthly. Respondents aged 25-54 who transitioned from unemployment to full-time employment at least once.

Table D.11: Robustness: Effect of Duration on Reemployment Wages

	(1)	(2)	(3)	(4)
log duration	-0.002**	-0.002*	-0.002*	5.55e-04
	(7.34e-04)	(8.76e-04)	(0.001)	(0.001)
Ind. FE	N	N	N	N
R^2	.276	.292	.277	.341
N	14,505	4,266	3,008	1,926

Notes: SIPP Sample, 1/1996-11/2013. Respondents aged 25-54 who transitioned from unemployment to full-time employment, with an unemployment spell up to one year. Controls are listed in Table 1. Column 1 reports results for the OLS regression allowing for left-censored spells, column 2 excludes recalls but includes workers with durations up to 52 weeks, column 3 reports results for the sub-sample of males, and column 4 reports results for the sub-sample of females. ***, **, and * indicate significance at the 1%, 5%, and 10% levels. Robust standard errors are reported in parentheses.

Table D.12: Robustness: Effect of Duration on the Job-Finding Probability

	(1)	(2)	(3)	(4)
duration	-0.032***	-0.003**	-0.033***	-0.030***
	(0.002)	(0.001)	(0.002)	(0.002)
$duration^2$	0.002***	2.88e-04***	0.002***	0.002***
	(1.46e-04)	(8.57e-05)	(1.73e-04)	(1.79e-04)
$duration^3$	-6.48e-05***	-9.76e-06***	-6.21e-05***	-5.48e-05***
	(4.26e-06)	(2.51e-06)	(4.88e-06)	(5.08e-06)
$duration^4$	5.96e-07***	1.00e-07***	5.56e-07***	4.94e-07***
	(4.09e-08)	(2.41e-08)	(4.55e-08)	(4.79e-08)
Ind. FE	N	N	N	N
R^2	.326	.756	.609	.622
N	73,635	52,407	30,501	23,897

Notes: SIPP Sample, 1/1996-11/2013. Respondents aged 25-54 who transitioned from unemployment to full-time employment, with an unemployment spell up to one year. Controls are listed in Table 1. Column 1 reports results allowing for left-censored spells, column 2 excludes recalls but includes workers with durations up to 52 weeks, column 3 reports results for the sub-sample of males, and column 4 reports results for the sub-sample of females. ***, **, and * indicate significance at the 1%, 5%, and 10% levels. Robust standard errors are reported in parentheses.

Table D.13: Effect of Duration on Reemployment Wages, Weighted

	(1)	(2)	(3)
duration	-0.002*	-0.002**	-0.004
	(0.001)	(0.001)	(0.003)
UI benefit		3.24e-04***	
		(4.39e-05)	
HH assets		4.67e-04***	
		(1.71e-04)	
HH income		2.50e-05***	
		(4.53e-06)	
Ind. FE	N	N	Y
R^2	.317	.349	.006
N	3,807	3,807	300

Notes: SIPP Sample, 1/1996-11/2013. Respondents aged 25-54 who transitioned from unemployment to full-time employment (excluding recalls). Controls are listed in Table 1. Column 1 reports results for the OLS regression of workers at all durations, column 2 is the same regression with controls for unemployment insurance, household assets, and household income, and column 3 includes individual fixed effects with 1-digit industry and occupation dummies due to the smaller sample size. All regressions are weighted using longitudinal panel weights. ***, **, and * indicate significance at the 1%, 5%, and 10% levels. Robust standard errors are reported in parentheses.

Table D.14: Effect of Duration on the Job-Finding Probability, Weighted

	(1)	(2)	(3)	(4)
duration	-0.039***	-0.036***	-0.007	-0.035***
	(0.003)	(0.003)	(0.008)	(0.002)
duration^2	0.003***	0.003***	0.002***	0.003***
	(2.25e-04)	(2.23e-04)	(6.90e-04)	(2.12e-04)
duration^3	-8.24e-05***	-7.88e-05***	-7.16e-05***	-7.90e-05***
	(6.54e-06)	(6.49e-06)	(2.09e-05)	(6.59e-06)
duration^4	7.69e-07***	7.40e-07***	7.56e-07***	7.53e-07***
	(6.29e-08)	(6.25e-08)	(2.09e-07)	(6.63e-08)
UI benefit		-6.23e-05***		
		(5.05e-06)		
HH assets		-1.04e-05		
		(7.83e-06)		
HH income		4.93e-06***		
		(5.66e-07)		
Ind. FE	N	N	Y	N
R^2	.013	.020	.038	.018
N	44,089	44,089	1,210	44,089

Notes: SIPP Sample, 1/1996-11/2013, monthly. Respondents aged 25-54. Controls are listed in Table 1. Column 1 reports results for the OLS regression of workers at all durations, column 2 is the same regression with controls for unemployment insurance, household assets, and household income, and column 3 includes individual fixed effects with 1-digit industry and occupation dummies due to the smaller sample size. Column 4 reports marginal effects from a probit regression, using the same controls as in column 1. All regressions are weighted using longitudinal panel weights. ***, **, and * indicate significance at the 1%, 5%, and 10% levels. Robust standard errors are reported in parentheses.

E Fixed Home Productivity

This section presents the estimation of the model in which all workers have the same, constant, home productivity $\overline{h}$. Tables E.15 and E.16 summarize the parameters and targeted moments used in the estimation. Because the UN rate is not pinned down but the NE rate is pinned down in the fixed-h model, there is a trade-off between the level of the labor force participation rate and the NE rate. In particular, the lower is $\underline{p}$, the lower are both NE and participation. In order to obtain the best fit of the targets, the fixed h model implies a far lower probability of skill appreciation while employed but a higher probability of skill loss while unemployed, relative to the full model. In turn, this leads to a smaller change in the job-finding probability with duration, shown in Table E.17. Simultaneously, the large rate of skill loss leads to a change in the log reemployment wage similar to that in the full model. Figure 2 shows the full path of the job-finding probability and reemployment wages with duration, labeled "fixed h model". Finally, Table E.18 corresponds to Table 9 and shows the implied business-cycle moments.

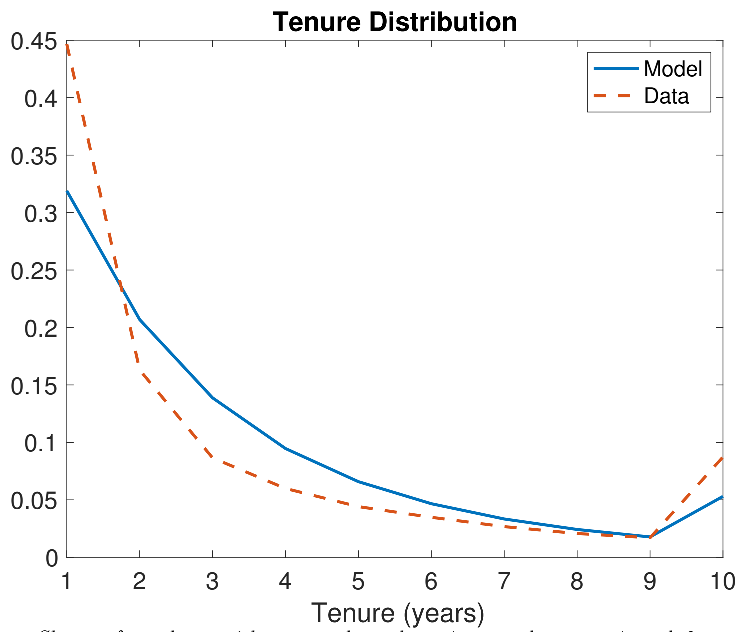
Table E.15: Parameters

Parameter	Value	Description
β	.9959	Discount factor
ξ	.0021	Death probability
γ	.4	job-finding probability $p(\theta) = min\{\theta^\gamma, 1\}$
$\underline{p}$	.139	Labor force participation cutoff
λ_e	.248	Frequency of job search, employed
k	21.4	Vacancy cost
Δ	.091	Log step size, home and market skills
σ_y	.111	Standard deviation, match-specific productivity
δ	.016	Separation probability
h_1	.696	Unique home skill
z_1	.410	Lowest market skill
π_{Ez}	.313	$z' = \min\{z + \Delta, z_0\}$ with prob π_{Ez} if E, z otherwise
π_{Eh}	0	$h' = \max\{h - \Delta, h_1\}$ with prob π_{Eh} if E, h o.w.
π_{Uh}	0	$h' = \min\{h + \Delta_h, h_{10}\}$ with prob π_{Uh} if U, h o.w.
π_{Uz}	.618	$z' = \max\{z - \Delta_z, z_1\}$ with prob π_{Uz} if U, z o.w.
χ	.756	Workers' bargaining power

Table E.16: Targets: Fixed Home Skill

Description	Target	Model [95% CI]
Annual interest rate	5%	5%
Average working lifetime	40 years	40 years
Matching function elasticity w.r.t v	.4	.4
NE Rate (Quarterly)	.053	.058 [.053, .066]
EE Rate (Monthly)	.014	.014 [.013, .016]
UE Rate (Quarterly)	.228	.225 [.210, .237]
EU Rate (Quarterly)	.016	.016 [.015, .017]
UN Rate (Quarterly)	.192	.027 [.021, .033]
Relative value of non-market work	.71	.81 [.802, .820]
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	0.73% [-1.0%, 3.2%]
Average Skill Depreciation (Annual)	.20	.23 [0, .56]
Tenure Distribution	See Figure A6	

Figure A6: Tenure Distribution, Fixed h Model



Notes: Share of workers with tenure less than 1 year, between 1 and 2 years, ..., greater than 10 years. Dashed line shows the empirical tenure distribution in the fixed h model for workers between 23 and 55 in the SIPP, solid line shows the distribution computed from model simulations.

Table E.17: Untargeted Moments, Steady State, Fixed h Model

Description	Data	Model
% change, log reemployment wage	−0.5%	−9.8%
% change, job-finding probability	−53.8%	−22.7%
Unemployment rate	4.2%	5.9%
Labor force participation rate	84%	92%
Initial job-finding probability (1 month)	.145	.239

Table E.18: Business Cycle Moments

x	std(x)	$corr(x, Y)$	$corr(x_t, x_{t-1})$	std(x)/std(Y)
EU, data	.089	-.63	.59	8.78
EU, model	.034	-.09	.70	3.35
UE, data	.088	.76	.75	8.68
UE, model	.038	-.03	.76	3.75
UN, data	.106	.61	.62	10.46
UN, model	.096	-.02	.64	9.42
NE, data	.103	.52	.38	10.16
NE, model	.052	-.09	.57	5.08
N, data	.003	.21	.69	.30
N, model	.006	-.37	.99	.59
U, data	.117	-.84	.93	11.55
U, model	.028	-.16	.93	2.71

F Homogeneity Under Proportional Vacancy Costs

This section describes an alternative specification of one of the model's key equations such that all equilibrium value and policy functions depend only on one relative skill, denoted $\sigma = z/h$. For simplicity I assume that there is no on-the-job search and normalize $y = 1$ for all matches. Extending this section to the case with heterogeneous match-specific productivity is straightforward.

Define the growth rate of skill z between two consecutive periods t and $t+1$ as $g_{z,t+1} = \frac{z_{t+1}}{z_t}$, and the growth rate of h as $g_{h,t+1} = \frac{h_{t+1}}{h_t}$. The realizations of g_z and g_h are contained in the compact sets $G_z = [\frac{\underline{z}}{\bar{z}}, \frac{\bar{z}}{\underline{z}}]$ and $G_h = [\frac{\underline{h}}{\bar{h}}, \frac{\bar{h}}{\underline{h}}]$. Assume that the growth rates in the current period depend only on the current employment state such that each period, $(g_z, g_h) \sim \Pi_i$ where $\Pi_i : G_z \times G_h \to [0,1]$ for $i = U, E$.

The laws of motion for skill growth implicitly define the Markov transition functions for their levels. Given initial skill levels $(z_0, h_0) \sim F_0$, the skill levels in any period t are given by $(z_t, h_t) = (g_z z_{t-1}, g_h h_{t-1})$ where $(g_z, g_h) \sim \Pi_U$ if the worker is unemployed at the beginning of period t and $(g_z, g_h) \sim \Pi_E$ if she is employed. These *iid* growth shocks implicitly define the laws of motion Q_U and Q_E for the levels of (z, h).

Timing in the extended model is identical to the benchmark, with the only difference being the cost of vacancy posting. Here, this cost is proportional to the productivity of the worker type present in a given submarket, kz, where $k > 0$ is a constant.

F.1 Homogeneity in the Planner's Problem

The planner's problem may be written as

$$W(\psi) = \sup_{\theta \in \mathbb{R}_+} \left\{ C(\theta|\psi) + \beta \mathbb{E} \max_{d \in [\delta,1]} \{ W(\hat{\psi}) \} \right\} \tag{A9}$$

subject to the laws of motion for u and e given by (4) and (5), and where aggregate consumption in a period is given by

$$C(\theta|\psi) \equiv \int_X hu(x) + Aze(x) - kz\theta u(x) \; dx \tag{A10}$$

A result similar to Theorem (1) holds in the extended model, with the planner's component value functions given by

$$\begin{aligned} W_U(z,h,A) = \max_{\theta \in [0,\bar{\theta}]} \Bigg\{ & h - kz\theta + \beta(1-\xi) \Big[\mathbb{E}_U(W_U(z',h',\hat{A})|z,h,A) \\ & + p(\theta)\mathbb{E}_U\big(W_E(z',h',\hat{A}) - W_U(z',h',\hat{A})|z,h,A\big) \Big] + \beta\xi\mathbb{E}_0(W_U(z',h',\hat{A})|A) \Bigg\} \end{aligned} \tag{A11}$$

$$W_E(z,h,A) = Az + \beta(1-\xi)\mathbb{E}_E\Big(\max_{d\in[\delta,1]} \big\{d(h' + \beta D(z',h',\hat{A},W_U)) \\ + (1-d)W_E(z',h',\hat{x})\big\}|z,h,A\Big) + \beta\xi\mathbb{E}_0(W_U(z',h',\hat{A})|A) \quad \text{(A12)}$$

One can write the relative skill of a worker of type (z,h) as $\sigma = \frac{z}{h}$, where $\sigma \in \mathcal{S}$, with $\mathcal{S} = [\frac{\underline{z}}{\bar{h}}, \frac{\bar{z}}{\underline{h}}]$. Finally, let $\tilde{Q}_i,\ i = U, E$, denote the stationary transition function for σ, and $\tilde{F}_0$ denote the exogenous distribution from which newborn workers' relative skills are drawn, and denote the associated expectation operators $\tilde{\mathbb{E}}_i$, $i = U, E, 0$, respectively. The distributions of workers across employment states and relative skills are denoted $\tilde{u}$ and $\tilde{e}$. Using this notation, Assumption 1 implies that σ and A are independent.

Proposition 2. *Equations (A11) and (A12) are homogeneous in the sense that* $h\mathrm{w}_i(\sigma, A) \equiv hW_i(\sigma, 1, A)$, $i = \{U, E\}$, *where*

$$\mathrm{w}_U(\sigma,A) = \max_{\theta\in[0,\bar{\theta}]} \Bigg\{1 - ks\theta + \beta(1-\xi)\Big[\tilde{\mathbb{E}}_U(g'_h \mathrm{w}_U(\sigma',\hat{A})|\sigma,A) \\ + p(\theta)\tilde{\mathbb{E}}_U\big(g'_h(\mathrm{w}_E(\sigma',\hat{A})) - \mathrm{w}_U(\sigma',\hat{A}))|\sigma,A\big)\Big] + \beta\xi\tilde{\mathbb{E}}_0(g'_h \mathrm{w}_U(\sigma',\hat{A})|A)\Bigg\}$$

$$\mathrm{w}_E(\sigma,A) = As + \beta(1-\xi)\tilde{\mathbb{E}}_E\big(g'_h \max_{d\in[\delta,1]} \big\{d(1 + \beta D(\sigma',\hat{A},\mathrm{w}_U)) \\ + (1-d)\mathrm{w}_E(\sigma',\hat{A})\big\}|\sigma,A\big) + \beta\xi\tilde{\mathbb{E}}_0(g'_h \mathrm{w}_U(\sigma',\hat{A})|A)$$

where

$$\tilde{D}(\sigma,A,\mathrm{w}_U) = (1-\xi)\tilde{\mathbb{E}}_U\big(g'_h \mathrm{w}_U(\sigma',\hat{A})|\sigma,A\big) + \xi\tilde{\mathbb{E}}_0(g'_h \mathrm{w}_U(\sigma',\hat{A})|A)$$

The proof of this proposition can easily be shown by guess and verify, and is therefore omitted for brevity. Denote the optimal policies corresponding to the value functions w_U and w_E by $\theta^*(\sigma, A)$ and $d^*(\sigma, A)$.

F.2 Proportional Vacancy Costs in the Decentralized Economy

In the two-skill model, submarkets are indexed by (x, z, h, ψ), where $x \in \mathbb{R}$ is the value in terms of the worker's lifetime utility of the match and (z, h) is the type of worker for which the vacancy is intended. A firm may post a vacancy by paying a cost proportional to the worker's effective labor in the current period, kz, with $k > 0$. Each vacancy in a submarket offers the same value x, and firms commit to this value and the type of worker they will hire if a match occurs.

The value function for an unemployed worker of type (z, h) is

$$V_U(z,h,\psi) = \sup_x \Big\{ h + \beta(1-\xi)\Big[(1-p(\theta(x,z,h,\psi)))\mathbb{E}_U(V_U(z',h',\hat{\psi})|z,h,\psi) + p(\theta(x,z,h,\psi))x\Big] + \beta\xi\mathbb{E}_0(V_U(z',h',\hat{\psi})|\psi)\Big\} \quad \text{(A13)}$$

where the policy function is denoted $x(z, h, \psi)$ and the implied market tightness is denoted $\theta(x, z, h, \psi)$.

The value of the employed worker can be written

$$V_E(z,h,\psi;w) = w+\beta(1-\xi)\mathbb{E}_E(\max_{d\in[\delta,1]} \big\{d(1+D(z',h',\hat{\psi},V_u))+(1-d)V_E(z',h',\hat{\psi};w')\big\}|z,h,\psi) + \beta\xi\mathbb{E}_0(V_U(z',h',\hat{\psi})|\psi) \quad \text{(A14)}$$

where

$$D(z,h,\psi,V_U) = (1-\xi)\mathbb{E}_U\big(V_U(z',h',\hat{\psi})|z,h,\psi\big) + \xi\tilde{\mathbb{E}}_0(V_U(z',h',\hat{\psi})|\psi)$$

and the value for the firm given the expected separation probability d' is given by:

$$J(z,h,\psi;w) = Az - w + \beta(1-\xi)\mathbb{E}_E((1-d')J(z',h',\hat{\psi};w')|z,h,\psi) \quad \text{(A15)}$$

Combining (A14) and (A15) under the assumption of complete contracts gives the following match value:

$$V_M(z,h,\psi) = Az+\beta(1-\xi)\mathbb{E}_E(\max_{d\in[\delta,1]} \big\{d(1+D(z',h',\hat{\psi},V_u))+(1-d)V_M(z',h',\hat{\psi};w')\big\}|z,h,\psi) + \beta\xi\mathbb{E}_0(V_U(z',h',\hat{\psi})|\psi) \quad \text{(A16)}$$

where the policy function is denoted $d(z, h, \psi)$. Finally, the free entry condition may be written as:

$$kz \geq \beta(1-\xi)q(\theta(x,z,h,\psi))(\mathbb{E}_U(V_M(z',h',\hat{\psi})|z,h,\psi) - x(z,h,\psi)) \quad \text{and} \quad \theta(x,z,h,\psi) \geq 0 \quad \text{(A17)}$$

F.2.1 Homogeneity in the Decentralized Economy

Guess the following functional forms: $hV_U(\sigma, 1, A) \equiv hv_U(\sigma, A)$, $hV_E(\sigma, 1, A) \equiv hv_E(\sigma, A)$, where $\sigma = \frac{z}{h}$ is the relative skill of the worker of type (z, h). The value for the unemployed worker may be written

$$hv_U(\sigma,\psi) = \sup_x \Big\{ h + \beta(1-\xi)\Big[(1-p(\theta(x,z,h,\psi)))\tilde{\mathbb{E}}_U(h'v_U(\sigma',\hat{\psi})|z,h,\psi) + p(\theta(x,z,h,\psi))\tilde{x}\Big] + \beta\xi\tilde{\mathbb{E}}_0(h'v_U(\sigma',\hat{\psi})|\psi)\Big\}$$

Canceling terms and replacing $\frac{h'}{h}$ with g_h',

$$v_U(\sigma,\psi) = \sup_x \Big\{ 1 + \beta(1-\xi)\Big[(1-p(\theta(x,\sigma,\psi)))\tilde{\mathbb{E}}_U(g_h'v_U(\sigma',\hat{\psi})|\sigma,\psi) + p(\theta(\tilde{x},\sigma,\psi))\frac{x}{h}\Big] + \beta\xi\tilde{\mathbb{E}}_0(g_h'v_U(\sigma',\hat{\psi})|\psi)\Big\}$$

Similarly, using the guesses for the functional forms of V_U and V_M, the value of a match is given by

$$v_M(\sigma,\psi) = As + \beta(1-\xi)\tilde{\mathbb{E}}_E(g_h' \max_{d\in[\delta,1]}\big\{d(1+\tilde{D}(\sigma',\hat{\psi},v_U)) + (1-d)v_M(\sigma',\hat{\psi};w')\big\}|\sigma,\psi) + \beta\xi\tilde{\mathbb{E}}_0(g_h'v_U(\sigma',\hat{\psi})|\psi) \quad \text{(A18)}$$

where

$$\tilde{D}(\sigma,\psi,v_U) = (1-\xi)\mathbb{E}_U\big(g_h'v_U(\sigma',\hat{\psi})|\sigma,\psi\big) + \xi\mathbb{E}_0(g_h'v_U(\sigma',\hat{\psi})|\psi)$$

Finally, the transformed free entry condition is given by

$$ks \geq \beta(1-\xi)q(\theta(x,z,h,\psi))(\tilde{\mathbb{E}}_U(g_h'v_M(\sigma',\hat{\psi})|\sigma,\psi) - \frac{x(z,h,\psi)}{h}) \quad \text{and} \quad \theta(x,z,h,\psi) \geq 0$$

Solving for $x(z,h,\psi)$ when $\theta(x,\sigma,\psi) > 0$, and plugging into the transformed value of unemployment gives:

$$v_U(\sigma,\psi) = \sup_\theta \Big\{ 1 - ks\theta + \beta(1-\xi)\Big[(1-p(\theta))\tilde{\mathbb{E}}_U(g_h'v_U(\sigma',\hat{\psi})|\sigma,\psi) + p(\theta)\tilde{\mathbb{E}}_U(g_h'v_M(\sigma',\hat{\psi})|\sigma,\psi)\Big] + \beta\xi\tilde{\mathbb{E}}_0(g_h'v_U(\sigma',\hat{\psi})|\psi)\Big\}$$

Note that the equation above can also be written as the combination of the following two equations:

$$v_U(\sigma,\psi) = \sup_{\tilde{x}} \Bigg\{ 1 + \beta(1-\xi)\Bigg[(1 - p(\theta(\tilde{x},\sigma,\psi)))\tilde{\mathbb{E}}_U(g_h' v_U(\sigma',\hat{\psi})|\sigma,\psi) + p(\theta(\tilde{x},\sigma,\psi))\tilde{x} \Bigg] + \beta\xi\tilde{\mathbb{E}}_0(g_h' v_U(\sigma',\hat{\psi})|\psi) \Bigg\} \quad \text{(A19)}$$

$$ks \geq \beta(1-\xi)q(\theta(\tilde{x},\sigma,\psi))(\tilde{\mathbb{E}}_U(g_h' v_M(\sigma',\hat{\psi})|\sigma,\psi) - \tilde{x}(\sigma,\psi)) \quad \text{and} \quad \theta(\tilde{x},\sigma,\psi) \geq 0 \quad \text{(A20)}$$

Defining $\tilde{x} = \frac{x}{h}$, equations (A19), (A18), and (A20) depend only on σ since g_h is independent of its past realizations, verifying the guess.

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