Skill Accumulation in the Market and at Home

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May 4, 2020

Abstract

An evolving outside option is introduced into a stochastic directed search model with skill loss during non-employment. Using multi-spell data from the SIPP, I show that average reemployment wages are only mildly sensitive to unemployment duration while the job finding probability is highly sensitive to duration, with evidence of true duration dependence in both variables. Though untargeted, the model produces a quantitatively accurate decline in the job finding probability and starting wage, improving over a model with a fixed outside option. The addition of aggregate shocks leads to an nonlinear response of the unemployment and participation rates during and after recessions, with more severe recessions resulting in stronger hysteresis.

Keywords: Non-employment, Job Search, Human Capital, Home Production
JEL Classification: E24, E32, J24, J64

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1 Introduction

Recent empirical evidence shows that reemployment and reservation wages are only mildly sensitive to unemployment duration (Fernandez-Blanco and Preugschat 2018, Krueger and Mueller 2016, Schmieder et al. 2016), despite the well-documented fact that the job-finding probability falls greatly with duration (Van den Berg and Van Ours 1996, Ahn and Hamilton 2019, among others). Much of the existing literature cannot jointly explain these facts, leading to the question of how to theoretically reconcile them.

This paper aims to answer this question by considering an evolving outside option for job-seekers. In the model, a worker's outside option behaves like a non-employment "skill": when employed, they learn on the job and when non-employed, their value of non-market activities increases with duration. The longer a worker is non-employed, the lower are their skills in market work and the higher is their value of non-market activities, decreasing the attractiveness of employment. These two forces amplify the decline in the job-finding probability with duration, but offset in the determination of their reemployment wages.

This paper contributes to the literature in several ways. Empirically, I use the Survey of Income and Program Participation (SIPP) to document the stylized facts above, previously found in other data sources such as the Current Population Survey (CPS). The SIPP allows me to control for fixed worker heterogeneity and thus changes in the composition of unemployed workers with duration.

Theoretically, I build a model with skill accumulation in the value of the outside option, which is interpreted broadly to include many plausible utility-generating activities during non-employment.[1] The model builds on Menzio and Shi (2011) by introducing evolving worker-specific in addition to constant match-specific productivity. Workers are ex-ante heterogeneous and each is endowed with an observable non-market and market skill that evolve stochastically conditional on their employment status. The model framework generates both "true" duration dependence, through changes in individual skills that affect each

[1]Differently from the home production literature, e.g, Benhabib et al. 1991 and Greenwood and Hercowitz 1991,"home skills" in this model are not fixed nor common across all workers.

worker's job-finding probability and wage, and through composition effects that drive changes in the average job-finding probability and reemployment wage across workers.

I estimate the model to match key moments in the data, including transition rates between employment states that identify the parameters driving skill accumulation. Though untargeted, the model succeeds in reproducing the key stylized facts in the steady state, and produces unemployment and labor force participation rates similar to the data.

In addition to steady state moments, the model is tractable enough to analyze business cycle dynamics of employment stocks and flows, which it matches well. The quantitative model implies a persistent decrease in participation and sluggish recovery of unemployment in response to temporary recessions. If the productivity shock is large enough, workers will leave the labor force: by strictly preferring to participate in non-market activities, some workers will permanently discontinue their job search. Because of skill changes, the model endogenously gives rise to hysteresis, resulting in a persistent fall in the participation rate and persistent rise in unemployment for years after the recession ends.

This paper is connected to a large literature on the link between worker characteristics and unemployment. Similar to standard models of learning by doing (e.g, Ljungqvist and Sargent 1998), the skill that a worker uses in production appreciates and the skill not in use depreciates. Such models with skill loss, and those with informational frictions in the spirit of Blanchard and Diamond (1994) imply that both reservation wages and job-finding probabilities should fall with duration.[2] A related literature studies earnings losses relative to measures of counterfactual earnings had a worker never lost her job. For instance, Huckfeldt (2016) shows that the earnings losses associated with job displacement documented by Davis and von Wachter (2011) and others are concentrated among workers who switch occupations. Differently, my focus is on the effect of duration, rather than the incidence of displacement itself, on reemployment earnings.

[2] In a related paper, Frijters and van der Klaauw (2006) consider the outside option of non-participation for unemployed workers in a model of skill loss. However, their model suggests a sharply declining reservation wage while workers are actively unemployed, at odds with evidence shown here and in Krueger and Mueller (2016) and Fernandez-Blanco and Preugschat (2018).

Models of hysteresis and persistence of high-unemployment episodes by Sterk (2016) and Schaal and Taschereau-Dumouchel (2019) are also highly related. The goal of this paper is to understand how a model of job search can match the steady state patterns in reemployment probabilities and wages; in addition I show that it can match several business cycle properties of the labor market which are the focus of Sterk (2016) and Schaal and Taschereau-Dumouchel (2019).

In complementary work, Fernandez-Blanco and Preugschat (2018) construct a model of firm ranking of workers to rationalize the same stylized facts that I study here. In their paper, workers are heterogeneous in fixed, unobservable productivities. Their model generates true duration dependence in the job-finding probability because firms' rankings of workers depend on duration through its role as a signal of a worker's type. Differently, my model considers the case when true duration dependence originates from changes in workers' characteristics. I document new evidence for this true duration dependence by controlling for individual fixed effects using SIPP data.

The paper proceeds as follows. Section 2 describes the motivating empirical evidence. Section 3 describes the model, beginning with the social planner's problem followed by the decentralized economy. Section 4 outlines the method for the estimation and reports the quantitative results both in and out of steady state. Section 5 concludes.

2 Motivating Evidence

Using CPS data, Fernandez-Blanco and Preugschat (2018) find that the average job-finding probability is roughly 50% lower after 12 months relative to 1 week of unemployment, while average hourly reemployment wages fall by less than 4% over the same horizon. I explore the robustness of these patterns using an alternative data source, the Survey of Income and Program Participation (SIPP), which allows me to control for composition effects present in the cross section stemming from unobservable, fixed worker heterogeneity.

I use the 1996 through 2008 SIPP panels to run regressions along the lines of Fernandez-Blanco and Preugschat (2018). The SIPP has several advantages over the CPS in addition to its multi-year coverage of individuals. First, it includes monthly observations of income, making it possible to study the starting wages of all workers reporting unemployment-to-employment (UE) transitions. Further, individuals in the SIPP are asked about their participation in government assistance programs, including unemployment insurance, allowing me to control directly for these benefits.

I consider prime-age workers – those between 25 and 54 years old – who transition into full-time employment following an unemployment spell, excluding the self-employed, workers who are in part- or full-time education and the retired. I identify the weekly unemployment duration by considering only full spells that begin while a worker is in the sample. Further details on construction of the sample and robustness checks are contained in Online Appendix D.

Table 1 contains estimates of the following regression on the sample of workers who made a UE transition over two consecutive months:

$$\ln(w_{i,t}) = \beta_0 + \beta_1 dur_{i,t-1} + \gamma X_{it} + \delta_t + \mathbb{1}_{FE}\alpha_i + \varepsilon_{it} \tag{1}$$

where i and t denote the individual and date (month-year), $w_{i,t}$ denotes real non-imputed monthly earnings before deductions, $dur_{i,t-1}$ denotes the duration of unemployment in weeks, X_{it} are individual characteristics, and δ_t are month-year fixed effects.[3] Finally, α_i is an individual fixed effect which is included only in the fixed effect regression, denoted by $\mathbb{1}_{FE}$ and shown in column (3) of the table. Individual characteristics X_{it} in all specifications include a quadratic term for age, dummies for gender, marital status, education, race, children, state, industry and occupation in the previous job, whether the worker switched industry or occupation from the previous job, the reason for unemployment, and a dummy if the reference month is the last month of each wave.[4,5] Finally, all

[3]Earnings rather than wages are used because earnings are directly reported in the survey, whereas wages are computed using reported hours which may increase measurement error.

[4]Though the coefficient is not reported in Table 1, the results are in line with the conclusion of Huckfeldt (2016) that workers who switch industry or occupation after an unemployment spell have significantly lower earnings than workers who do not switch.

[5]The dummy if the reference month is the last month of each wave aims to control for the well-known seam bias in the SIPP. See Ham et al. 2009 for further discussion.

regressions include the aggregate unemployment rate in month $t-1$. Additional covariates included in some specifications are discussed below.

Table 1: Effect of Duration on Reemployment Earnings

	(1)	(2)	(3)
duration	-7.84e-04	-9.85e-04	-0.007**
	(8.42e-04)	(8.19e-04)	(0.003)
UI benefit		3.11e-04***	
		(4.17e-05)	
HH assets		4.76e-04***	
		(1.22e-04)	
HH earnings		3.00e-05***	
		(3.62e-06)	
Ind. FE	N	N	Y
R^2	.288	.320	.017
N	4,934	4,934	351

Notes: SIPP Sample, 1/1996-11/2013. Respondents aged 25-54 who transitioned from unemployment to full-time employment, with an unemployment spell up to one year. Controls include a quadratic term for age, the unemployment rate, gender, marital status, dummies for education, race, children, state, 1-digit industry and occupation prior to the unemployment spell, whether the worker switched industry or occupation, reason for unemployment, year, month, and a dummy for the last reference month in each wave. Column 1 reports results for the OLS regression of workers at all durations up to 52 weeks, column 2 is the same regression with controls for unemployment insurance, household assets, and household earnings, and column 3 includes individual fixed effects. ***, **, and * indicate significance at the 1%, 5%, and 10% levels. Robust standard errors are reported in parentheses.

The first column of Table 1 contains the baseline estimates. The second column includes all of the regressors in the first column plus the amount of unemployment benefit the worker received in the last month before the UE transition occurred, household assets, and household earnings. The third column includes individual fixed effects, using only those workers who experienced two UE transitions during the time they were included in the panel.[6]

The results in the first two columns of Table 1 show that pooling across

[6] Although the sample on which I am able to perform the fixed effects regression is small, in Online Appendix D I show that these workers are similar to the full population in several observable characteristics.

individuals, there is no significant effect of duration on reemployment earnings. The baseline estimates reported in column 1 show that the coefficient is not statistically different from zero. In the second column, unemployment benefits and intra-household insurance measured by assets and earnings raise the starting earnings of new hires out of unemployment. After controlling for these factors, the estimated effect on duration again is not statistically significant. Differently, column 3 shows that when controlling for unobserved fixed effects the resulting effect of duration is significantly negative: for workers with multiple UE spells, an additional week of unemployment duration decreases starting earnings by 0.8%. This estimate, however, should be interpreted with caution as it relies on a small sample of workers making at least two UE transitions over the roughly four-year period covered by the survey.

Table 2 contains the results from a linear probability model estimating the effect of unemployment duration on the job-finding rate. Specifically, columns 1 through 3 of the table contain estimates of the following regression on the sample of workers who were unemployed in the previous month:

$$UE_{t-1,t} = \beta_0 + \beta_1 dur_{i,t-1} + \beta_2 dur_{i,t-1}^2 + \beta_3 dur_{i,t-1}^3 + \beta_4 dur_{i,t-1}^4 + \gamma X_{it} + \delta_t + \mathbb{1}_{FE}\alpha_i + \varepsilon_{it} \tag{2}$$

where $UE_{t-1,t}$ is a dummy equal to one if the worker found a job between month $t-1$ and t, and $dur_{i,t-1}$, X_{it}, δ_t, and α_i are defined as in (1). Columns 1 through 3 report the resulting estimates using the same regressors as in Table 1, with the addition of higher order terms in duration following Fernandez-Blanco and Preugschat (2018). Column 4 reports the marginal effects from a probit regression including the same regressors in column 1.

The estimates in Table 2 show the strong negative relationship between duration and the job-finding probability. As in Table 1, after controlling for observables, the effects of benefits and household insurance significantly affect the job-finding probability with the expected sign. The estimates suggest that an additional week of unemployment duration is associated with between a 1% and 3% fall in the job-finding probability.

Most importantly for the model proposed in this paper, the estimates in column 3 of Tables 1 and 2 provide evidence of true duration dependence in

Table 2: Effect of Duration on the Job-Finding Probability

	(1)	(2)	(3)	(4)
duration	-0.032***	-0.031***	-0.013***	-0.021***
	(0.002)	(0.002)	(0.004)	(0.001)
duration^2	0.002***	0.002***	0.001***	0.002***
	(1.25e-04)	(1.25e-04)	(3.43e-04)	(1.00e-04)
duration^3	-5.91e-05***	-5.76e-05***	-4.18e-05***	-4.27e-05***
	(3.54e-06)	(3.53e-06)	(1.06e-05)	(3.30e-06)
duration^4	5.30e-07***	5.18e-07***	4.10e-07***	3.98e-07***
	(3.32e-08)	(3.30e-08)	(1.07e-07)	(3.44e-08)
UI benefit		-1.99e-05***		
		(2.72e-06)		
HH assets		-1.11e-05*		
		(5.97e-06)		
HH earnings		2.10e-06***		
		(3.18e-07)		
Ind. FE	N	N	Y	N
R^2	.613	.614	.006	.115
N	54,398	54,398	1,548	50,206

Notes: SIPP Sample, 1/1996-11/2013. Respondents aged 25-54 with an unemployment spell up to one year. Controls are listed in Table 1. Column 1 reports results for the OLS regression of workers at all durations up to 52 weeks, column 2 is the same regression with controls for unemployment insurance, household assets, and household earnings, and column 3 includes individual fixed effects. Column 4 reports marginal effects from a probit regression, using the same controls as in column 1. ***, **, and * indicate significance at the 1%, 5%, and 10% levels. Robust standard errors are reported in parentheses.

both the job-finding rate and reemployment earnings. At the same time, because of the small number of multi-spell workers, I cannot rule out the fact that unobserved heterogeneity explains some of the observed negative duration dependence. Overall, the evidence summarized here differs greatly from the behavior of reservation and reemployment earnings in many job search models. In the next section, I propose a model that can rationalize both the within-worker and cross-sectional results shown above.

3 Model

The model extends Menzio and Shi (2011) to include stochastically evolving worker-specific skills.[7] The economy is populated by a continuum of workers of measure 1 and a continuum of firms with a positive measure. Time is discrete and the horizon is infinite. All agents are risk neutral and discount the future at rate $\beta \in (0,1)$. There is a single consumption good produced in the economy. Workers are ex ante heterogeneous and defined by their observable skills in the market and at home, respectively (z,h), where $z \in Z = [\underline{z}, \overline{z}]$ and $h \in H = [\underline{h}, \overline{h}]$. The value of h reflects the worker's outside option and encompasses both home production and leisure activities.[8]

The range of values the market and home skills may take satisfy $0 < \underline{z} < \overline{z} < \infty$ and $0 < \underline{h} < \overline{h} < \infty$. Workers' skills evolve each period depending on their current employment status: employed (E) or unemployed (U).[9] The stationary transition functions for these processes are given by Q_U and Q_E, where $Q_i(s,s')$ is the probability that a worker in employment state $i \in \{U, E\}$ with skill pair $s \equiv (z,h)$ today transitions to skills $s' = (z', h')$ next period. Denote the set of skill pairs $S = Z \times H$.

Firms maximize their present value of profits and operate a constant returns to scale technology that turns 1 unit of inelastically supplied labor from a worker with skills (z,h) in a match with productivity y into Azy units of output. Aggregate productivity A is common to all firms and lies in the set $\mathbf{A} = [\underline{A}, \overline{A}]$ where $0 < \underline{A} < \overline{A} < \infty$. Each period, A is drawn from a distribution denoted by the stationary transition function $P(\cdot, A)$. Match-specific productivity y lies in the set $Y = [\underline{y}, \overline{y}]$, drawn from the distribution F when a firm and worker meet. Unmatched workers are unemployed, and produce h units of output through home production.

Timing is as follows. At the beginning of the period, nature draws aggregate

[7]Though the paper aims to explain two facts about UE transitions, I allow for on-the-job search to improve the quantitative fit of the model.

[8]Like Campbell and Ludvigson (2001), I consider time not working as productive time, and therefore do not make a distinction between leisure and home work.

[9]In the model, I refer to all non-employed workers as unemployed. The distinction between workers who are unemployed and out of the labor force depends on the definition of active job search and will be made quantitatively in Section 4.

productivity A and each worker's skills (z, h) given their employment status the previous period.[10] Workers face an exogenous probability of death, $\xi \in (0,1)$. I assume that workers have a bequest motive whereby they derive utility from future generations of newborn workers. The same mass ξ of workers is born each period with skills drawn from the stationary distribution F_0.

The period then proceeds in the following stages: production, separation, search and matching. During the production stage, employed worker-firm pairs and unemployed workers produce. In the separation stage, with probability $d \in [\delta, 1]$ an employed worker separates from his match and enters unemployment, where $\delta \in (0,1)$ is the exogenous separation probability.

The labor market is defined by submarkets in which workers and vacancy-posting firms meet. The cost of posting one vacancy is a strictly positive constant k. Unemployed workers can search for jobs with probability 1. Employed workers have the ability to search with probability λ_e.

During the matching stage, the number of hires in a submarket is determined by a constant returns to scale technology $m(a, v)$ where a is the number of applicants in the submarket and v is the number of vacancies. Market tightness θ is a function of the ratio of vacancies to applicants in a given submarket. The probability that a vacancy meets a worker is $q(\theta) \equiv \frac{m(a,v)}{v}$, where $q : \mathbb{R}_+ \rightarrow [0,1]$ is a twice continuously differentiable, strictly decreasing and convex function with $q(0) = 1$, and $q'(0) < 0$. Similarly, the probability that a worker meets a vacancy is given by $p(\theta) = q(\theta)\theta$, where $p : \mathbb{R}_+ \rightarrow [0,1]$ is twice continuously differentiable, strictly increasing and strictly concave with $p(0) = 0,\ p(\infty) = 1$, $p'(0) < \infty$, and $p'(q^{-1}(\cdot))$ concave. When a firm and worker meet, nature draws the match-specific productivity y, which is immediately observable, and the pair decides whether to enter into the match. Define $c_u : S \times Y \rightarrow [0,1]$ ($c_e : S \times Y \times Y \rightarrow [0,1]$) as the probability that a meeting between an unemployed (employed) worker and a firm becomes a match given the worker's skills and match-specific productivity (and match-specific productivity in an employed worker's current job).

[10] This assumption guarantees that learning by doing occurs only after the first period of production. Similar theoretical results are obtained with alternative specifications for skill evolution.

Each period, the aggregate state is given by $\psi \equiv (A, u, e)$, with the set of possible values that ψ may take denoted by Ψ. The first element of ψ is the aggregate productivity $A \in \mathbf{A}$. The second element is a function $u : S \rightarrow [0, 1]$, describing the distribution of unemployed workers across skills at the beginning of the production stage, where $u(z, h)$ denotes the mass of workers who are unemployed with skills (z, h). The third element is a function $e : S \times Y \rightarrow [0, 1]$, where $e(z, h, y)$ denotes the mass of employed workers with skill (z, h) and match-specific productivity y at the beginning of the production stage.

3.1 Planner's Problem

The planner's problem is to maximize aggregate consumption by choosing how to allocate workers and vacancies across submarkets. Specifically, in the separation stage the planner chooses separation probabilities for each employed worker. In the search stage, the planner chooses how many vacancies firms post in each submarket and in which submarkets workers search. Separated workers must spend one period in unemployment before searching and new hires must produce for one period in employment before separating.

At the beginning of each period, the planner observes aggregate state ψ and chooses θ for each submarket, job acceptance probabilities c_u and c_e, and d for each worker-firm pair. Since workers with a given skill paid are identical, the planner chooses one strategy for all workers with the same skills in each employment state. Due to two-dimensional worker heterogeneity, the planner may find it optimal to assign the same market tightness to two types of workers. However, it is equivalent in terms of welfare to create two skill pair-specific submarkets with the same tightness. Therefore, I assume that there is one submarket per skill pair in each period. Aggregate consumption given the planner's choices and the aggregate state is equal to the sum of production of all workers less any vacancy costs:

$$C(d,\theta_u,\theta_e|\psi) \equiv \int_S \Bigg(\big(h - k\theta_u(s|\psi)\big)u(s|\psi) \\ + \int_Y \big(Azy - k(1-d(s,y|\psi))\lambda_e\theta_e(s,y|\psi)\big)e(s,y|\psi)dy \Bigg) ds$$

where u and e denote the distributions of unemployed and employed workers in this period's production stage, θ_u and θ_e are vectors of market tightness for the unemployed and employed, and d is the vector of separation probabilities.

The planner's problem is to solve:

$$W(\psi) = \max_{d,\theta_u,\theta_e,c_u,c_e} \big\{ C(d,\theta_u,\theta_e|\psi) + \beta \mathbb{E} W(\hat{\psi}) \big\} \tag{3}$$

where c_u is the vector of job acceptance probabilities for the unemployed, with $c_u(s,y|\psi)$ denoting the probability that an unemployed worker s matches with a firm with productivity y, and c_e is the vector of job acceptance probabilities for the employed, with $c_e(s,y',y|\psi)$ denoting the probability that an employed worker s currently with match productivity y' matches with a firm with productivity y. The planner solves (3) subject to the endogenous laws of motion for u and e, given by the following expressions:

$$\hat{u}(s|\psi) = \xi f_0(s) + (1-\xi)\Bigg[\int_Y \int_S \big[1 - p(\theta_u(s'|\psi))c_u(s',y|\psi)f(y)\big] u(s'|\psi) Q_U\big(ds',s\big)dy \\ + \int_Y \int_S d(s',y|\psi)e(s',y|\psi)Q_E\big(ds',s\big)dy \Bigg] \tag{4}$$

$$\hat{e}(s,y|\psi) = (1-\xi)\Bigg[f(y)\int_S p(\theta_u(s'|\psi))c_u(s',y|\psi)u(s'|\psi)Q_U\big(ds',s\big) \\ + \int_Y \int_S (1-d(s',y|\psi))\big[1-\lambda_e p(\theta_e(s',y|\psi))c_e(s',y,y'|\psi)\big]e(s',y|\psi)Q_E\big(ds',s\big)f(y')dy' \\ + \lambda_e f(y) \int_Y \int_S (1-d(s',y'|\psi))p(\theta_e(s',y'|\psi))c_e(s',y',y|\psi)e(s',y'|\psi)Q_E\big(ds',s\big)dy' \Bigg] \tag{5}$$

The mass of workers with a given skill s next period is equal to the mass of workers transitioning to s tomorrow using transition functions Q_U and Q_E, given their skills today. Henceforth, for any aggregate variable, a caret denotes its value at the beginning of next period's production stage.

Equation (4) says that the distribution of unemployed workers with skill s at the beginning of next period is given by a constant mass of newborn workers plus those surviving workers who transition to skill s who were either unemployed this period and did not match with a firm, or who were employed and separate from their matches at the beginning of next period. Equation (5) is similar and gives the mass of surviving workers who will be employed at the beginning of next period's production stage with skills s. Note that the planner takes the distribution of newborns f_0 as given. The following assumption is made for tractability.

Assumption 1. *(i) Productivity s evolves independently from A. For any $B \subseteq S$, $C \subseteq \mathbf{A}$, $b \in S$ and $c \in \mathbf{A}$,*

$$Pr\big(s' \in B, \hat{A} \in C \big| s = b, A = c, i\big) = Q_i(b, B)P(c, C) \quad \textit{for } i \in \{E, U\}$$

(ii) The following monotonicity conditions hold:

if $f : A \to \mathbb{R}$ is any non-decreasing integrable function,

$$\textit{then } \int f(\hat{A})P(A, d\hat{A}) \textit{ is also nondecreasing}$$

if $g : S \to \mathbb{R}$ is any non-decreasing integrable function,

$$\textit{then } \int g(s')Q_i(s, ds') \textit{ is also nondecreasing, for } i = U, E$$

Part (i) of Assumption 1 states that given a worker's skill pair today, her current employment state determines its evolution. Aggregate productivity is assumed to be independent of worker-specific skills to exclude human capital externalities as in the seminal work of Lucas (1988). Part (ii) imposes some minimal structure on the transition functions Q_E, Q_U and P, and will be used in the results regarding monotonicity in Theorem 1. Informally, the latter states that workers with higher skills in the current period can expect to have higher skills next period.

3.1.1 Constrained Efficiency

The above formulation of the planner's problem leads to the following theorem. All proofs are left to Online Appendix B.

Theorem 1. *(i) The following problem is equivalent to (3).*

$$\tilde{W}(\psi) = \int_Z \int_H \left(W_U(z,h,A)u(z,h) + \int_Y W_E(z,h,y,A)e(z,h,y)\, dy \right) dh\, dz \quad (6)$$

where

$$\begin{aligned} W_U(z,h,A) = \max_{\theta_u \in [0,\overline{\theta}]} \Bigg\{ h - k\theta_u + \beta \Bigg[\xi \mathbb{E}_0[W_U(z',h',\hat{A})] \\ + (1-\xi)\Bigg(\int_Y \max_{c_u \in [0,1]} \big\{ p(\theta_u)c_u \mathbb{E}_U[W_E(z',h',y,\hat{A})] \\ + (1-p(\theta_u)c_u)\mathbb{E}_U[W_U(z',h',\hat{A})] \big\} f(y)dy \Bigg) \Bigg] \Bigg\} \end{aligned} \quad (7)$$

$$\begin{aligned} W_E(z,h,y,A) = \max_{\theta_e \in [0,\overline{\theta}], d \in [\delta,1]} \Bigg\{ Azy - (1-d)k\lambda_e\theta_e + \beta \Bigg[\xi \mathbb{E}_0[W_U(z',h',\hat{A})] \\ + (1-\xi)\Bigg(d\mathbb{E}_E[W_U(z',h',\hat{A})] + (1-d)\Bigg(\int_Y \max_{c_e \in [0,1]} \big\{ \lambda_e p(\theta_e)c_e \mathbb{E}_E[W_E(z',h',y',\hat{A})] \\ + (1-\lambda_e p(\theta_e)c_e)\mathbb{E}_E[W_E(z',h',y,\hat{A})] \big\} f(y')dy' \Bigg) \Bigg) \Bigg] \Bigg\} \end{aligned} \quad (8)$$

with $0 < \overline{\theta} < \infty$ is a constant. (ii) $\tilde{W}(\psi)$ is the unique solution to (6). (iii) W_U is strictly increasing in h and weakly increasing in z and A, and W_E is strictly increasing in z, y, and A and weakly increasing in h if Assumption 1 holds. (iv) The policy correspondences θ^*, c_e^*, c_u^*, and d^* associated with (6) depend on ψ only through A: $\theta_u^*(z,h,\psi) = \theta_u^*(z,h,A)$, $\theta_e^*(z,h,y,\psi) =$

$\theta_e^*(z,h,y,A)$, $c_e^*(z,h,y,y',\psi) = c_e^*(z,h,y,y',A)$, $c_u^*(z,h,y',\psi) = c_u^*(z,h,y',A)$ *and* $d^*(z,h,\psi) = d^*(z,h,A)$.

The expectation operator $\mathbb{E}_i$ denotes the expectation taken with respect to the transition function Q_i, $i \in \{U,E\}$ for the worker's skills conditional on the current skills and aggregate state, e.g. $\mathbb{E}_E(W_U(z',h',\hat{A})) \equiv \mathbb{E}_E(W_U(z',h',\hat{A})|z,h,A) = \int_{\underline{A}}^{\overline{A}} \int_S W_U(s',\hat{A}) Q_E(s,ds') P(A,d\hat{A})$. Similarly, the expectation operator $\mathbb{E}_0$ denotes the expectation taken with respect to the aggregate state and distribution F_0.

Several elements of the model complicate the analysis of the planner's problem relative to those analyzed in the previous literature. In particular, the planner's decisions in terms of market tightness and separation probabilities affect the endogenous distributions of worker skills across employment states in a nontrivial way. Unlike models with *iid* draws of match-specific productivity only (e.g. Menzio and Shi 2011), here the persistence of workers' productivity when transitioning between employment states causes the planner's choices to not only affect the level of employment, but also to dynamically affect the distribution of skills across employment and unemployment. This distributional dependence interacts with the uncertainty about the aggregate productivity.

However, as Theorem 1 shows, the planner's objective of maximizing aggregate consumption is equivalent to maximizing the consumption of each skill type separately. Intuitively, the law of large numbers implies that the matching and separation probabilities exactly determine the endogenous distributions over worker skills next period. Since aggregate consumption is the sum of consumption of each skill type, the planner's policy correspondences are identical when maximizing the aggregate utilities jointly or maximizing each skill type's utility separately.

3.1.2 Results

I now solve for the planner's optimal choices of market tightness, match acceptance probabilities, and separation probabilities from (7) and (8). Beginning with the job acceptance probability for the unemployed, c_u^*, after observing the state and match productivity y, the optimal acceptance probability for an unemployed worker with skill pair (z,h) is

$$c_u^*(z,h,y,A)=\begin{cases}1 & \text{if } \mathbb{E}_U\big[W_E(z',h',y,\hat{A})-W_U(z',h',\hat{A})\big]\geq 0\\ 0 & \text{otherwise}\end{cases}$$

Note that because of the timing in the model, the worker accepts the match today but does not produce until next period. Although match-specific productivity is observable and fixed for the duration of the match, the planner must take the expectation of the evolution of the workers' skills and the aggregate productivity when deciding whether to form the match. If the expected value of employment exceeds the expected value of unemployment, it is optimal for the unemployed worker to accept.

Similarly, the job acceptance probability for an employed worker with skill pair (z,h) and current match-specific productivity y who meets a firm with productivity y' is

$$c_e^*(z,h,y,y',A)=\begin{cases}1 & \text{if } \mathbb{E}_E\big[W_E(z',h',y',\hat{A})-W_E(z',h',y,\hat{A})\big]\geq 0\\ 0 & \text{otherwise}\end{cases}$$

If the expected value of the new job exceeds the expected value of remaining in the current match, it is optimal for the employed worker to make a job-to-job transition. Following Menzio and Shi (2011) job acceptance probabilities c_e^* and c_u^* can be represented by reservation productivity r_e^* and r_u^*, where $c_e^*(z,h,y,y',A)=1$ if $y'\geq r_e^*(z,h,y,A)$ and $c_u^*(z,h,y',A)=1$ if $y'\geq r_u^*(z,h,A)$, and zero otherwise.

Next, the optimal separation probability $d^*(z,h,y,A)=1$ if

$$\begin{aligned}&k\lambda_e\theta_e^*(z,h,y,A)+\beta(1-\xi)\mathbb{E}_E\big[W_U(z',h',\hat{A})\big]\\ &\geq \beta(1-\xi)\bigg(\int_Y \lambda_e p(\theta_e^*(z,h,y,A))c_e^*(z,h,y,y',A)\mathbb{E}_E\big[W_E(z',h',y',\hat{A})\big]f(y')dy'\\ &\quad+\int_Y[1-\lambda_e p(\theta_e^*(z,h,y,A))c_e^*(z,h,y,y',A)]\mathbb{E}_E\big[W_E(z',h',y,\hat{A})\big]f(y')dy'\bigg)\end{aligned}$$

otherwise $d^*(z,h,y,A) = \delta$. The benefit for the planner to separate a match is the saved vacancy costs this period, since newly separated workers cannot search, plus the discounted expected value of unemployment. The cost of separating for the planner is the contribution of the employed worker to aggregate output, either by remaining in the same match or making a job-to-job transition. If the benefit from separating strictly exceeds the cost, the planner will send all workers with skills (z,h) in matches y to unemployment when the aggregate productivity is A. Note that because employed workers can search with probability λ_e while the unemployed can search with probability 1, it may be optimal to send workers with low match-specific productivity to unemployment even after many periods in a match.

Finally, the first order condition for market tightness for unemployed workers with skills (z,h) implies the following complementary slackness condition

$$k \geq \beta(1-\xi)p'(\theta_u^*(z,h,A)) \int_Y c_u^*(z,h,y,A)\mathbb{E}_U\big[W_E(z',h',y,\hat{A}) - W_U(z',h',\hat{A})\big] f(y)dy \tag{9}$$

and $\theta_u^*(z,h,A) \geq 0$.

Similarly, for the employed workers, we have

$$\begin{aligned} k \geq \beta(1-\xi)p'(\theta_e^*(z,h,y,A)) \int_Y c_e^*(z,h,y,y',A)\mathbb{E}_E\big[&W_E(z',h',y',\hat{A}) \\ &- W_E(z',h',y,\hat{A})\big] f(y')dy' \end{aligned} \tag{10}$$

and $\theta_e^*(z,h,y,A) \geq 0$. Because workers may not survive to begin the job, the right hand side of the two equations above is discounted by $\beta(1-\xi)$.

Intuitively, (9) implies that if $\mathbb{E}_U(W_E - W_U)$ is decreasing as h increases and z decreases, then θ_U^* and therefore the job-finding probability $p(\theta_U^*)$ are decreasing. In general, because both terms are affected in opposite ways by the changes in the two skills, one would expect some non-monotonicity in the job-finding probability at the individual worker level. To be able to quantitatively evaluate the distributional implications of the model for the job-finding probability and re-employment wages as functions of duration, it is necessary to solve the model numerically. In Section 4 I quantitatively evaluate the model's predictions for the stylized facts discussed in Section 2 and its ability to replicate key business cycle patterns by solving the planner's problem. Before discussing the quantitative exercise, I briefly turn to the decentralized economy to address wages.

3.2 Decentralized Economy

As is standard in directed search models, it can be shown that the decentralized equilibrium, assuming bilaterally efficient contracts, is block recursive in the sense that agents' value and policy functions are independent of the distributions of workers, and is efficient. However, wage setting in the decentralized economy directly effects the stylized facts that this model set out to rationalize. In the remainder of the paper, I assume that wages are set by Nash Bargaining. As long as wage contracts can be conditioned on all current and past states (worker, firm, and aggregate), it follows that wages do not affect match creation or destruction. The discussion of the decentralized economy and the resulting equilibrium is contained in Online Appendix A and the proof of efficiency can be found in Online Appendix B.

3.3 Discussion of the Mechanism

This model provides one explanation for the empirical pattern shown in Section 2. Although the model introduces a simple force, that is changes in the outside option that depend on a worker's employment state, the forces at play that generate such a pattern are complex. For an individual worker, the job finding probability is given by the free entry condition (9), which depends on a worker's expected gain from entering employment, given the evolution of her skills. Theorem 1 states that both W_U and W_E are increasing in both skills, thus the sign of the expected gain from entering employment is ambiguous. Moreover, because workers of different skills enter unemployment at the same time, the pool of unemployed at a given duration will be heterogeneous, affecting the path of the average job finding probability as a function of duration. The next section shows that the inclusion of the evolving outside option allows this model to qualitatively match the facts documented in Section 2, while standard models with skill loss or informational frictions, such as the large literature based upon Ljungqvist and Sargent (1998) or Blanchard and Diamond (1994), cannot account for this pattern under the assumption that workers value non-employment homogeneously.

I interpret the outside option broadly, which could incorporate family insurance, technological progress in leisure activities, or home work that otherwise would be outsourced. All of these features contribute to the outside option of non-employed workers in this model.[11,12]

There are several alternative mechanisms one could consider to explain the stylized facts presented here. If workers retrain during unemployment, it is observationally equivalent to their market skills decreasing by less with duration, leading to lower average skill loss. In order to match the average skill loss estimated by Keane and Wolpin (1997), targeted in the next section to pin down the parameter driving market skill depreciation, some workers would need to fail to retrain and therefore experience large amounts of skill loss and in turn large declines in their starting wages. Similarly, an increasing cost of search effort would be consistent with the falling job finding probability, but would decrease average wages at long durations as workers' outside options fall.

4 Quantitative Results

This section presents the quantitative results of the model. Section 4.1 discusses the estimation of model parameters. Sections 4.2 and 4.3 describe the steady state and business cycle implications of the quantitative model, respectively.

[11] A worker's outside option during non-employment can reflect insurance from family members; Kaplan (2012) and Dyrda et al. (2012) suggest that moving in with one's parents during economic downturns is an important form of insurance for young adults. In addition, Aguiar et al. (2017) find that young, unskilled men's leisure time, mostly that devoted to playing video games, has increased nearly one-for-one with the decline in hours of market work since the mid-2000s.

[12] In Online Appendix D.1 I provide additional evidence to support the plausibility of the model's main mechanism through which the outside option affects the duration of unemployment. In particular, I show that exogenous shocks to the outside option coming from changes in weather can lead to changes in the allocation of time in non-market activities.

4.1 Estimation

The model is estimated to match moments in US data, and is solved in steady state, fixing the aggregate productivity A to 1.[13] The length of a period is one month. Several parameters are chosen exogenously. The discount factor β implies an annual interest rate of 5%. The probability of death ξ is chosen such that the expected lifetime of a worker is 40 years and the distribution from which newborn workers' skills are drawn, F_0, is equal to the ergodic distribution of unemployed, u. Following Menzio and Shi (2011), the functional form for the probability that a worker matches with a firm is given by $p(\theta) = \min\{\theta^{\gamma}, 1\}$. The matching function parameter γ is set to 0.4.

The final 12 parameters in Table 3 are estimated jointly using the simulated method of moments, by simulating the model in steady state for 20,000 workers over 500 months and discarding the first 50 periods.[14] Model moments, shown in Table 4, correspond to model averages over the simulations with their 95% confidence intervals in brackets.

The state spaces for individual skills z and h are discretized into grids of 10 points with equally log-spaced state vectors where $\log(s_{i+1}) - \log(s_i) = \Delta$, for $i = 1, \dots 9,\ s = \{z, h\}$. I assume that match-specific productivity y is log-normal with mean zero and standard deviation σ_y, and discretize its realizations over 10 grid points using the Tauchen (1986) method.

The transition matrices for individual skills are defined as follows. When a worker is employed and has market skill z, with probability π_{Ez} the worker will move up one rung on the skill ladder, and with probability $1 - \pi_{Ez}$ her skill does not change. Similarly, the probability that an unemployed worker's home skill increases by one step is π_{Uh}, and the probability it remains the same is $1 - \pi_{Uh}$. Movements down the skill ladder are defined symmetrically, and written formally below:

[13] The model presented here has a unique equilibrium, but as with other models of human capital accumulation, may give rise to multiple steady states. Nonetheless, model simulations with a reasonable probability of death (implying a working lifetime of 30 or 40 years) suggest that the steady state is unique.

[14] These parameters provide the best fit of an unweighted average of all targeted moments summarized in Table 4. Due to the high dimensionality of the parameter vector, standard errors are omitted.

$$\log(z') \mid \text{E} = \begin{cases} \min\{\log(z_{10}), \log(z) + \Delta\} & \text{with probability } \pi_{Ez} \\ \log(z) & \text{otherwise} \end{cases}$$

$$\log(h') \mid \text{E} = \begin{cases} \max\{\log(h_1), \log(h) - \Delta\} & \text{with probability } \pi_{Eh} \\ \log(h) & \text{otherwise} \end{cases}$$

$$\log(h') \mid \text{U} = \begin{cases} \min\{\log(h_{10}), \log(h) + \Delta\} & \text{with probability } \pi_{Uh} \\ \log(h) & \text{otherwise} \end{cases}$$

$$\log(z') \mid \text{U} = \begin{cases} \max\{\log(z_1), \log(z) - \Delta\} & \text{with probability } \pi_{Uz} \\ \log(z) & \text{otherwise} \end{cases}$$

Table 3: Parameters

Parameter	Value	Description
β	.9959	Discount factor
ξ	.0021	Death probability
γ	.4	Job-finding probability $p(\theta) = min\{\theta^\gamma, 1\}$
$\underline{p}$	.156	Labor force participation cutoff
λ_e	.114	Frequency of job search, employed
k	11.6	Vacancy cost
Δ	.113	Log step size, home and market skills
σ_y	.094	Standard deviation, match-specific productivity
δ	.008	Separation probability
h_1	.770	Lowest home skill
z_1	.822	Lowest market skill
π_{Ez}	.743	$z' = \min\{z + \Delta, z_0\}$ with prob π_{Ez} if E, z otherwise
π_{Eh}	.351	$h' = \max\{h - \Delta, h_1\}$ with prob π_{Eh} if E, h o.w.
π_{Uh}	.491	$h' = \min\{h + \Delta_h, h_{10}\}$ with prob π_{Uh} if U, h o.w.
π_{Uz}	.560	$z' = \max\{z - \Delta_z, z_1\}$ with prob π_{Uz} if U, z o.w.

For comparability with the data, worker types with a matching probability greater (less) than $\underline{p}$ are considered unemployed (out of the labor force).

Henceforth, this threshold will be referred to as the "labor force cutoff". The moments discussed below refer to the unemployed as those workers with matching probabilities above this cutoff.

All transition rates are computed in the SIPP for the relevant sample. The job-to-job transition (EE) rate is expressed as the monthly rate; the remaining gross flows are computed at a quarterly frequency. In the model, the EE rate is computed as the number of employed workers accepting a new job each period, and is pinned down by the probability that employed workers search, λ_e. The UE transition rate in the model is computed as the average share of workers who transition to employment within a quarter relative to the number of unemployed at the beginning of the quarter, and is driven by the vacancy cost, k. The value of this parameter suggests that the average vacancy cost is equal to 5 months' production in the average match. The somewhat high value of k relative to the literature is necessary to match both the relative value of non-market work and the UE rate, which both depend on the distribution of skills across workers. Next, the model implies that roughly half of all EU transitions are quits, driven by the non-linear effect of changes in home and market skills on workers' choices of submarket, and the higher efficiency of searching while unemployed. Together with the endogenous quit rate, the exogenous separation rate, δ pins down the EU rate. The threshold for the meeting probability, $\underline{p}$, below which workers are considered out of the labor force pins down the NE flow. Notice that while the estimated value (.156) is high, workers choose whether or not to accept job offers based on the match-specific productivity which is drawn after the meeting occurs, thus, the job acceptance probability will in general be lower than the meeting probability given by $\underline{p}$. Lastly, the UN rate in the data pins down the speed at which workers exit active job search, which is driven by the speed at which they accumulate home skills while unemployed, π_{Uh}.

To pin down home skill loss during employment, π_{Eh}, consider two workers with the same market skill who have just entered unemployment. If one worker had experienced a longer employment spell than the other, on average her home skill will be lower due to depreciation, and therefore her job-finding probability will be higher since she has a lower value of remaining unemployed. In the data,

Table 4: Model Fit

Description	Target	Model [95% CI]
Annual interest rate	5%	5%
Average working lifetime	40 years	40 years
Matching function elasticity w.r.t v	.4	.4
NE Rate (Quarterly)	.053	.046 [.042, .051]
EE Rate (Monthly)	.014	.014 [.013, .016]
UE Rate (Quarterly)	.228	.228 [.210, .251]
EU Rate (Quarterly)	.016	.017 [.015, .018]
UN Rate (Quarterly)	.192	.205 [.178, .228]
Relative value of non-market work	.71	.71 [.674, .740]
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	0.76% [-1.8%, 2.0%]
Average Skill Depreciation (Annual)	.20	.12 [0, .55]
Tenure Distribution	See Figure 1	

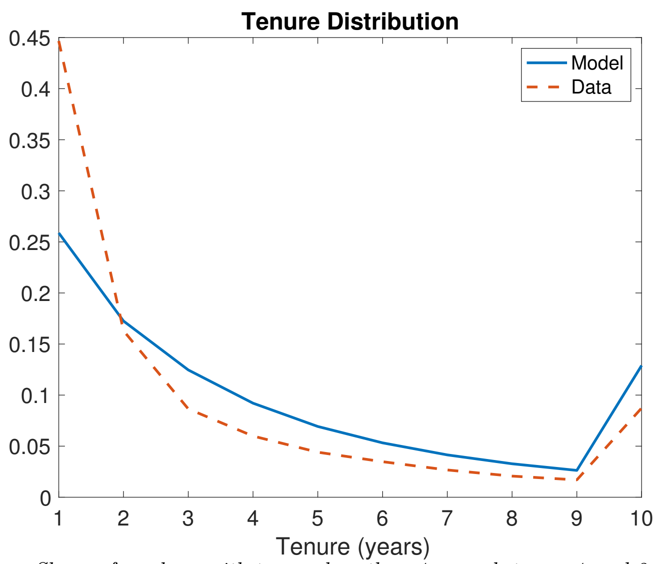
the change in the job-finding probability over tenure is the estimated marginal effect of an additional year of pre-unemployment tenure on the job-finding probability in the first month of unemployment, controlling for observables,[15] for workers with up to 15 years of tenure. This moment is equivalently computed in the model as the average change in the job-finding probability in the first

[15] This estimate comes from the CPS's Displaced Worker Survey (DWS). See Online Appendix C for details.

period of unemployment with an additional year of pre-unemployment tenure.

Market skill loss during unemployment, driven by π_{Uz}, is pinned down by the estimates of skill depreciation by Keane and Wolpin (1997), who find that in one year of unemployment, skills depreciate by 10% for blue collar workers and 30% for white collar workers. The target for the estimation is taken to be the average between these two values, 20%. In the model, this is computed as the average change in market skills over one year of unemployment.

Figure 1: Tenure Distribution



Notes: Share of workers with tenure less than 1 year, between 1 and 2 years, ..., greater than 10 years. Dashed line shows the empirical tenure distribution for workers between 25 and 54 in the SIPP, solid line shows the distribution computed from model simulations.

The value of z_1, step size Δ, speed of market skill accumulation π_{Ez}, and variance of the match-specific productivity distribution σ_y are chosen to minimize the distance to the empirical tenure distribution, following Menzio and Shi (2011). Intuitively, the distribution of market skills and match-specific productivity drives the duration of employment relationships.[16] The fit is shown in Figure 1.

[16]Alternatively, one could use returns to experience to pin down the speed of skill accumulation. However, wages are not pinned down due to efficiency of the equilibrium, therefore I do not use moments related to wages to pin down any of the parameters in Table 3.

Finally, the lowest home skill, h_1, is chosen such that the expected value of an unemployed worker's home productivity relative to an employed worker's market productivity in the steady state matches the estimate of the relative value of non-market to market activity in Hall and Milgrom (2008), $\frac{\mathbb{E}_U(h)}{\mathbb{E}_E(z)} = .71$.

Wages

By efficiency, the equilibrium outcomes are independent of wages. However, wages are key in matching one of the stylized facts motivating this paper. To discipline wages in the model, I assume Nash bargaining, with the worker's bargaining power pinned down by returns to market experience in the data. Returns to experience are estimated by Kambourov and Manovskii (2009) as the regression coefficient representing the annual return to experience in terms of real wages in the PSID between 1981 and 1992. In the model, the annual implied wage increase with experience is equal to the average annually compounded increase in wages over the cross section of new hires. The resulting estimate of bargaining power is 0.44, implying an annual return to experience of 2.3%, equal to the return found in the data.

Model Fit

Overall, the model fits the data well, with few exceptions. The simulated model implies that many workers have market skills close to the minimum value of z upon entry into unemployment; since depreciation is bounded by the value of z_1, average skill depreciation is therefore lower in the model than in the data, but remains within the range estimated by Keane and Wolpin (1997). The NE rate is slightly lower and the UN rate is slightly higher than their targets, implying a somewhat lower labor force participation rate, which is discussed further below. Finally, regarding the tenure distribution, Menzio and Shi (2011) find that the "inspection good" model of match-specific productivity underestimates the share of matches with short tenure, as is the case here. Incorporating a noisy signal on match quality that is realized only after producing once would improve the model fit in this dimension.

4.2 Steady State

Using the parameter values above, this section discusses the quantitative implications of the steady state model. I first discuss a set of untargeted moments. Then I show the model's fit of the stylized facts, and decompose the forces in the model driving these results.

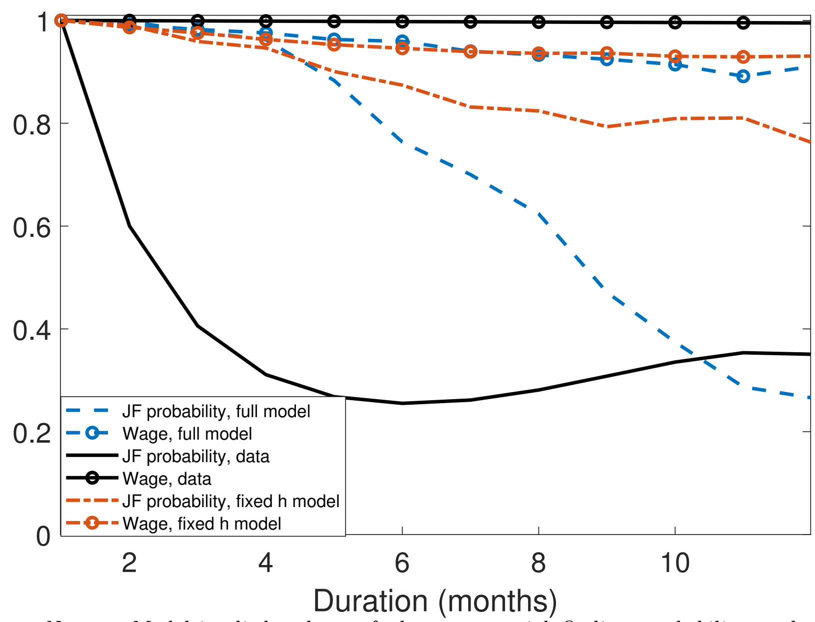
Table 5 summarizes several untargeted moments. All moments in the table are computed over the period 1996-2013 to be consistent with the analysis in Section 2. For comparison to the log wages in the data, wages are scaled such that the average reemployment wage in the model simulations is equal to average real log monthly earnings reported by prime-age workers in the SIPP. The change in the log reemployment wage is expressed as the percentage change in wages over the first year of unemployment, that is, the average wage for workers with 12 months of duration relative to the average wage for workers with 1 month of duration, the shortest possible duration observed in the model. The change in the job-finding probability, shown in the second row of the table, is computed similarly. The model predicts a much larger decline in the job-finding probability than in the reemployment wage, as documented in the data in Section 2. The model-implied unemployment rate and labor force participation rates are very close to the SIPP averages for prime-age workers. Finally, the level of the job-finding probability after 1 month of duration in the model is higher than the level in the data conditional on observables estimated in Section 2.

Table 5: Untargeted Moments, Steady State

Description	Data	Model
% change, log reemployment wage	−0.5%	−10.1%
% change, job-finding probability	−53.8%	−73.4%
Unemployment rate	4.2%	3.8%
Labor force participation rate	84%	81%
Initial job-finding probability (1 month)	.145	.301

The aggregate job-finding probability and reemployment wage are drawn as dashed lines in Figure 2. The solid lines correspond to the predicted data controlling for observables from the SIPP.[17] For comparison, I re-estimate the model with a fixed home skill (the "fixed h" model) to match the relevant targets from Table 4; model-implied moments and estimated parameters are shown in Online Appendix E. The predictions of the fixed h model are shown with dot-dashed lines. Though untargeted, the full model generates a relatively mild decline in the average reemployment wage but a large drop in the job-finding probability, a large improvement over the fixed h model, which predicts a fall in the job-finding probability of similar magnitude to the decline in wages.

Figure 2: Normalized Job-Finding Probability and Reemployment Wage



Notes: Model-implied values of the average job-finding probability and log reemployment wages over unemployment duration, reported in months. Dashed lines indicate values in the full model, dot-dashed lines in the fixed h model, and solid lines reproduce the predicted values from the SIPP.

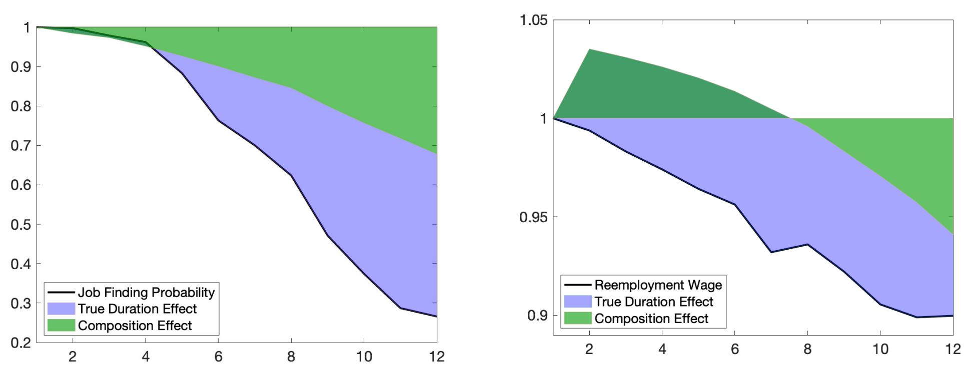
By losing one dimension of heterogeneity, the fixed h model requires vastly different parameter values for market skill changes than the full model.[18] In

[17] See Online Appendix D.3 for details.

[18] Tables of parameter values and targeted and untargeted moments are shown in Online

particular, because unemployment is less attractive due to the lack of home skill accumulation, the model requires a far higher vacancy posting cost to match the UE rate. This, in turn necessitates a higher frequency of job search for employed workers, and a higher probability of skill loss while unemployed. Finally, in order to best match the tenure distribution, the fixed h model requires a lower value of skill appreciation, π_{Ez}. As can be seen from the figure, the parameter values estimated for the fixed h model imply starting wages with a similar decline as wages the full model, but do not generate a sufficiently large fall in the job-finding probability.

Figure 3: Decomposition: Job-Finding Probability and Wage



Notes: Left panel: decomposition of the model-implied normalized job-finding probability. Right panel: decomposition of the normalized reemployment wage. The green shaded part (top) of the figure indicates the proportion of the decline due to composition, and the purple area (bottom) indicates the effect of true duration dependence. Composition accounts for 43% of the decline in the job-finding probability and 60% of the decline in the wage after 12 months of unemployment.

Next, I decompose the decline in the job-finding probability and reemployment wage into the true duration and composition effects, shown in Figure 3. The composition effect accounts for 44% of the decline in the job-finding probability after 12 months of unemployment. This result is lower than the recent empirical results of Ahn and Hamilton (2019) who find that the majority of the decline is due to composition changes. The composition effect in the model is computed as the average job-finding probability holding workers' skills constant at the initial values when entering unemployment. Results are different for the

Appendix E.

average reemployment wage shown in the right panel of Figure 3, with the composition effect accounting for 63% of the decline in wages after 12 months of unemployment. The composition effect alone would lead to higher wages in the first 7 months of the unemployment spell, as workers who remain unemployed tend to have higher home skills upon becoming unemployed, leading them to receive higher starting wages. Only at long durations, when the composition of workers has shifted towards those with lower market skills, does composition lead to a decline in wages.

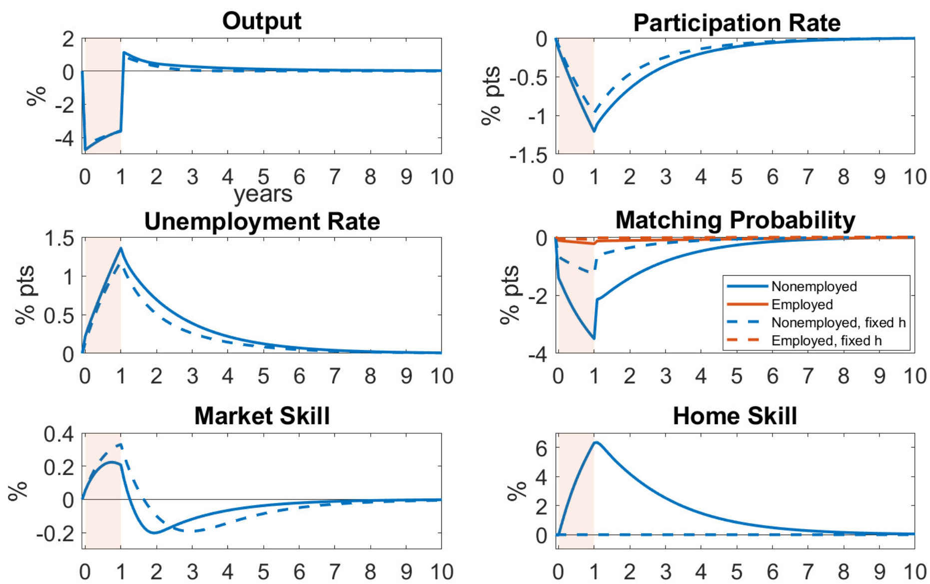
4.3 Business Cycles

In this section aggregate uncertainty is incorporated into the quantitative model through fluctuations in A. In the data, aggregate productivity is chosen to match the seasonally adjusted real average output per worker in the non-farm business sector constructed by the BLS. The process is discretized into a 5-state vector using the Tauchen (1986) method, where the highest and lowest states correspond to two standard deviations above and below the mean, respectively. Normalizing the average log productivity to zero, the vector of aggregate productivity is given by $A = [0.949 \quad 0.974 \quad 1.0000 \quad 1.026 \quad 1.053]$. I will refer to a recession (boom) as a prolonged period of aggregate productivity below (above) its mean.

I first study the effect of a 12-month recession, indicated by the shaded region in Figure 4. Tables 6 and 7 contain the simulated path of aggregate variables for 10 years following the recession in the full and fixed-h models, respectively. Figure 4 shows these paths graphically.

I first discuss the effects on the full model, shown by the solid lines in the figure. The effect on aggregate output is shown in the first row, first column. The negative shock causes the relative value of market work to fall, making home production more attractive. In the second column, the participation rate falls slightly on impact, reflecting the fact that some workers' optimal job finding probabilities drop below the labor force cutoff. The fall in participation continues over the course of the recession, as more unemployed workers leave the labor force as their skills evolve. Despite the decline in participation, the

Figure 4: Aggregate Response, 12 Month Recession



Notes: Responses of aggregate variables to a 12-month decline in aggregate productivity of two standard deviations below the mean. Responses measured in deviations from steady state ($A = 1$). Shaded region indicates the simulated recession.

panel in the second row, first column shows that more workers enter unemployment due to longer unemployment durations. The panel in the second row, second column shows that the job finding probability for both employed and unemployed workers falls on impact, with the probability for the unemployed declining by almost 4 percentage points by the end of the recession.

Because workers are less likely to leave unemployment, their skills have more time to evolve, shown in the last row of the figure. The left panel shows the path of the average market skill for employed workers, and the right panel shows the path of the average home skill for nonemployed workers. Interestingly, the average market skill rises during the recession because the workers with the lowest market skills are those who endogenously separate from their matches. When these workers enter nonemployment, their home skills begin to increase, shown in the right panel. This increase is far larger than the effect on average market skills because most newly unemployed workers have relatively low home skills, leaving room for large gains over time as their skills accumulate.

Table 6: Full Model, Change from Pre-Recession Value

Years	Output	Participation	U rate	$p(\theta_U)$	$p(\theta_E)$	z	h
0	-4.74%	-0.14 ppts	0.21 ppts	-1.40 ppts	-0.12 ppts	0.05%	-0.05%
1	-3.58%	-1.21 ppts	1.36 ppts	-3.50 ppts	-0.23 ppts	0.21%	6.32%
2	0.45%	-0.65 ppts	0.69 ppts	-1.41 ppts	-0.12 ppts	-0.20%	4.19%
4	0.17%	-0.20 ppts	0.22 ppts	-0.48 ppts	-0.08 ppts	-0.07%	1.48%
6	0.08%	-0.06 ppts	0.07 ppts	-0.16 ppts	-0.06 ppts	-0.02%	0.16%
8	0.04%	-0.02 ppts	0.02 ppts	-0.05 ppts	-0.04 ppts	-0.01%	0.05%
10	0.02%	-0.01 ppts	0.01 ppts	0.02 ppts	-0.02 ppts	0.00%	0.02%

Table 7: Fixed-h Model, Change from Pre-Recession Value

Years	Output	Participation	U rate	$p(\theta_U)$	$p(\theta_E)$	z	h
0	-4.48%	-0.10 ppts	0.13 ppts	-0.68 ppts	-0.05 ppts	0.05%	0.00%
1	-3.62%	-0.97 ppts	1.18 ppts	-1.27 ppts	-0.08 ppts	0.33%	0.00%
2	0.29%	-0.47 ppts	0.51 ppts	-0.36 ppts	-0.03 ppts	-0.10%	0.00%
4	0.01%	-0.14 ppts	0.14 ppts	-0.12 ppts	-0.02 ppts	-0.14%	0.00%
6	0.01%	-0.05 ppts	0.05 ppts	-0.04 ppts	-0.01 ppts	-0.05%	0.00%
8	0.01%	-0.02 ppts	0.02 ppts	-0.01 ppts	0.00 ppts	-0.02%	0.00%
10	0.00%	-0.01 ppts	0.01 ppts	0.00 ppts	0.00 ppts	0.01%	0.00%

Regarding the dynamics after the recession, it is changes in average market and home productivity that drives the hysteresis in labor force participation shown in the top right panel. Workers who choose non-participation continue to evolve so that many of them do not return to the labor force even after the recession ends. Only a small share of nonparticipants reenter the labor force immediately when the recession ends – those whose job finding probabilities were close to the cutoff – while the remainder of nonparticipants remains out of the labor force. The participation rate reverts as non-employed workers are reborn with new productivity, taking roughly 10 years to return to its mean. Though not shown in this figure, the larger is the decline in aggregate productivity, the larger is the decrease in labor force participation and more persistent is its level. Market skills of employed workers fall persistently below their pre-recession mean after the recession ends as nonemployed workers who have lost significant skills accept new jobs. Finally, output rises above its pre-recession level immediately as the costs of vacancy posting for nonemployed workers is persistently lower following the recession due to the persistence in the job-finding probability.

The dashed lines in Figure 4 show the responses in the fixed h model. Output follows a similar path in response to the same negative shock relative to the full model, though both unemployment and participation show a smaller reaction to the shock. These smaller magnitudes are driven by a much smaller change in the job-finding rate for nonemployed workers, driven only by losses in market-specific skills. Finally, market skills for the employed increase by more than in the full model, and take longer to revert to their pre-recession level, because the speed of market skill loss estimated in the fixed-h model is higher, and the speed of market skill accumulation is lower than in the full model.

Table 8: Share of Decline Due to Composition Effects

	Boom	Recession
Job-finding Probability	.49	.30
Reemployment Wage	.28	.36

Table 8 shows the decomposition of the job finding rate and reemployment wage in recessions and booms. The composition effect accounts for 49% of the decline in the job finding probability and 28% of the change in the starting wage in booms; in recessions composition accounts for 30% and 36%, respectively. The remainder of the declines in both variables are accounted for by true duration dependence. Thus, the model predicts a pro-cyclical composition effect for the job finding rate and a counter-cyclical composition effect for the wage. In their discussion of cyclical heterogeneity, Ahn and Hamilton (2019) find that the composition effect is larger in recessions as more of the "high type" workers in their model become unemployed, thus, composition is counter-cyclical. Differently, in this model workers have longer average unemployment spells in recessions, leading to a larger role for true duration dependence through changes in workers' skills. Since changes in these skills reinforce the fall in the job-finding probability but offset in determining the wage, the role of true duration dependence in recessions is larger for the probability, but smaller for the wage.

Table 9 compares the model's business cycle moments to the data. In the model, transition rates are the quarterly averages of monthly series. The empirical moments related to worker flows and stocks are taken from Krusell et al. (2017) and are defined similarly.[19]

Table 9: Business Cycle Moments

x	std(x)	$corr(x, Y)$	$corr(x_t, x_{t-1})$	std(x)/std(Y)
EU, data	.089	-.63	.59	8.78
EU, model	.040	.03	.45	3.98
UE, data	.088	.76	.75	8.68
UE, model	.058	.15	.64	5.64
UN, data	.106	.61	.62	10.46
UN, model	.056	.06	.73	5.56
NE, data	.103	.52	.38	10.16
NE, model	.049	.16	.57	4.81
N, data	.003	.21	.69	.30
N, model	.002	-.15	.91	.16
U, data	.117	-.84	.93	11.55
U, model	.041	-.09	.93	4.02
h, model	.013	.03	.99	1.25

The model does a fair job at capturing the standard deviation, serial correlation, and volatility relative to output of the series reported in the table. In particular, although the model moments are less volatile than in the data, the serial correlation of nearly all of the labor market flows are close to their empirical counterparts. In particular, the transition rates are all more volatile than output, and the unemployment rate is more volatile than the non-participation rate. The model's performance is lacking when comparing the comovement

[19]The business cycle moments are taken from Krusell et al. (2017) instead of the SIPP sample used above because of the longer time span of their data.

with output. In almost every case, the comovement is close to zero in the model, but often procyclical (with the exception of the EU and unemployment rates) in the data. The EU rate in the model is mildly procyclical because workers in low quality matches are more likely to quit in order to search from unemployment for a better match-specific productivity, given the higher search probability. The UE and NE rates are less procyclical than in the data because the composition of non-employed workers in booms is made up of workers with higher home skills, leading to lower job finding rates. For the same reason, the non-participation rate is countercyclical because more workers drop out of the labor force in recessions when the outside option of home production is more attractive relative to market work.

Table 10: Present Value of Earnings Losses

	Boom	Recession
Full Model	.062	.033
Fixed h Model	.116	.107
Data (Davis and von Wachter 2011)	.110	.186

Finally , Table 10 shows that the quantitative model with aggregate shocks falls short in its predictions of the present value of earnings losses following Davis and von Wachter (2011). They define average lifetime earnings losses as the loss in the present value of earnings for workers who experienced a mass layoff, relative to a counterfactual had the layoff not occurred. In the model, earnings losses are computed analogously as total wages over the 20 years following a displacement relative to the counterfactual wages were the workers never to have entered unemployment, for workers with at least 3 years of pre-displacement tenure. The table shows that the model has difficulty matching the variation in earnings losses as a function of the aggregate conditions at the time of displacement, a common shortcoming of "skill-ladder" models such as this.[20] The full model implies that in recessions earnings losses are lower because

[20]Huckfeldt (2016) shows that the countercyclicality of occupation switching, and the larger

employed workers are less likely to climb the job ladder due to lower job finding probabilities for the employed as well as unemployed. Because workers accumulate more home skills due to longer unemployment durations in recessions, they wait for better job offers before making a UE transition, and therefore catch up more quickly to their counterfactual earnings. The cyclical pattern is similar in the fixed h model, but with relatively little difference in the earnings losses in booms and recessions. Because workers unambiguously lose skills during unemployment, overall earnings losses are higher in both booms and recessions in the fixed h model.

The business cycle analysis presented in this section is highly related to Sterk (2016) and Schaal and Taschereau-Dumouchel (2019). The mechanisms in both of these papers generate multiple equilibria, one of which leads to permanently higher unemployment and lower output, providing a better fit of labor market data over the business cycle. Like my paper, both of these works generate hysteresis in unemployment. By looking at the business cycle through a directed search model, I can solve the model out of steady state, and allow for two dimensions of heterogeneity. Although I have not found parameters such that my model permanently enters a "bad" steady state, it is possible that with a large enough aggregate shock, the hysteresis shown in Figure 4 could be permanent.

5 Conclusion

This paper develops a model with an evolving outside option in non-market activities to address two facts: the insensitivity of reemployment wages and high sensitivity of the job finding probability to unemployment duration. The model's key mechanism is driven by the fact that during unemployment, workers' outside options evolve, affecting their job search strategies and employment outcomes.

Empirically, I show that there is evidence of true duration dependence in both the reemployment wage and job-finding rate, a result novel to the liter-

earnings losses of switchers, can explain a large share of the empirical finding of Davis and von Wachter (2011) shown in the table.

ature. Theoretically, I build a model that allows for uncertainty in individual heterogeneity through stochastic skill accumulation, and incorporates fluctuations in aggregate productivity. The quantitative model matches both the cross-sectional and within-worker facts better than alternatives lacking skill accumulation in the outside option, and can be used to understand labor market fluctuations over the business cycle. The model suggests that changes in workers' outside options during unemployment is an important force in generating the observed responses of aggregate labor market variables to shocks, and predicts a small but persistent change in unemployment and the labor force participation rate stemming directly from these changes.

Future research using data on consumption or the informal sector of the economy linked to the time spent outside of the formal labor market could shed more light on the relevance of the evolving outside option. Further, the model could be extended to study the inefficiencies that arise when home production is excluded from aggregate output, which would have very different policy implications than the present model, which is efficient. Finally, the life-cycle implications of a model such as this would suggest variation in skills and earnings dynamics of cohorts entering the labor market in booms and recessions.

Acknowledgments: I thank Laura Veldkamp, two anonymous referees, and the associate editor for comments and suggestions that greatly improved this paper. I also thank Guido Menzio, Facundo Piguillem, Fernando Alvarez, Marco Casiraghi, Martin Ellison, Javier Fernandez-Blanco, Basile Grassi, Francesco Lippi, Hamish Low, Claudio Michelacci, Andrea Pozzi, Ludo Visschers, David Wiczer, and numerous seminar and conference participants for helpful comments. Some of this research was carried out while I was at the University of Oxford and the University of Rome, Tor Vergata.

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